UNIVERSITY OF LONDON

GENERAL CERTIFICATE OF EDUCATION

Ordinary Level

SUMMER, 1957

PURE MATHEMATICS

(a) ARITHMETIC AND TRIGONOMETRY

TUESDAY, *June* 18. – *Morning*, 9.30 *to* 11.30

All necessary working must be shown

SECTION A

[*Answer* ALL *questions in this section*.]

1. (i) Simplify $\dfrac{3}{\frac{1}{5}\left(\frac{1}{5}-\frac{1}{6}\right)}$.

(ii) Divide $\dfrac{6\cdot 3}{0\cdot 35}$ by $\dfrac{7}{17\cdot 5}$.

(iii) Express £145 10*s*. as a percentage of £363 15*s*.

2. (i) If certain goods cost £9 6*s*, 8*d*. per ton what is the price of 19 cwt. 3 qr. 14 lb.?

(ii) What is the length of the side of a square which has the same area as a rectangle with sides 2 ft. 4 in. and 28 ft. 7 in.?

3. (i) A train, 160 ft. long, is travelling at 25 m.p.h. Find how many seconds it takes to pass completely over a bridge 720 ft. long.

(ii) A beam of timber 15 ft. long, 11 in. wide and 8 in. thick weighs 480 lb. What will be the weight of another beam of the same kind of timber $12\frac{1}{2}$ ft. long, 10 in. wide and $5\frac{1}{2}$ in. thick?

4. Rainfall on a flat rectangular roof 20 ft. by 11 ft. flows into a cylindrical tub of diameter 5 ft. Find, in inches, the increase in depth of the water in the tub caused by a rainfall of 0·12 in. (Take $\pi = 22/7$).

5. A cargo is valued by its owner at £2,945. For what value must he insure the cargo so that in case of loss he can recover his estimated value of the cargo and the insurance premium which is at the rate of 5 per cent. of the insured value?

6. A rectangle has sides 10 in. and 17 in. long. Calculate the acute angle between its diagonals.

Section B

[*Answer any* THREE *questions from this section*.]

7. An English car travelled 7,500 miles using 231 gallons of petrol at 4*s*. 8*d*. per gallon and 28 pints of oil at 10*s*. 6*d*. per gallon. A French car travelled 15,900 km. using 1,175 litres of petrol at 64 francs per litre and 19 litres of oil at 200 francs per litre. Which of the two cars ran more economically and by how much, to the nearest penny, per hundred miles?

[Assume £1 = 980 francs, 8 km. = 5 miles].

8. A uniform circular metal disc of diameter 15 in. weighs 12 lb. For the purpose for which it was intended its diameter must be $14\frac{3}{4}$ in. and its weight 11 lb. When its diameter is reduced to the required size it is still too heavy and it is proposed to reduce the disc to the required weight of 11 lb. by drilling through it a number of $\frac{1}{4}$ in. diameter holes. How many such holes must be drilled?

9. (i) A man borrows £100 and agrees to pay 5% per annum compound interest on the loan. How much does he owe at the end of one year and at the end of two years?

(ii) Calculate the sum of money which becomes \$441 at 5% per annum compound interest (*a*) after one year, (*b*) after two years.

(iii) A man borrows £1,640 which, together with the compound interest at 5% per annum, he is to repay by two equal annual instalments, the first of which is payable one year after borrowing the money. Calculate the value of each instalment.

10. A circle is inscribed in a square *ABCD* of side 10 in. and the diagonal *AC* meets the circle in the points *E* and *F*, *E* being nearer to *A*. Calculate (i) the area of the triangle *ADE*, (ii) the angle *EDF*.

11. From a point *P* on the same level as the base of a tower, the angle of elevation of the top of the tower is 32° 14′. At a point *Q*, 15 ft. vertically above *P*, the angle of depression of the base is 5° 12′. Find (i) the height of the tower, (ii) the angle subtended by *PQ* at the top of the tower.

UNIVERSITY OF LONDON

GENERAL CERTIFICATE OF EDUCATION

Ordinary Level

SUMMER, 1957

PURE MATHEMATICS

(b) ALGEBRA

MONDAY, June 24. – *Morning*, 9.30 *to* 11.30

All necessary working must be shown

SECTION A

[*Answer* ALL *questions in this section*.]

1. (i) If $x = 1$, $y = -2$, $z = -3$, find the value of $(3x - y)^2 \div z^3$.

(ii) Factorise $(2x + 1)^2 - (x - 2)^2$.

(iii) Solve the equation $3x^2 - 4x = 0$.

(iv) Solve the equation $4p^2 = 25$.

2. (i) Express as a single fraction in its simplest form

$$\frac{a-b}{a+b} - \frac{a+b}{a-b}.$$

(ii) Find the value of x if $\dfrac{x}{12} - \dfrac{5(x-1)}{3} = \dfrac{3}{4}$.

(iii) If $y = \dfrac{a - bx}{1 + ax}$, express x in terms of a, b and y.

(iv) Expand and simplify $(a^2 + ab + b^2)(a - b)$.

3. (i) Solve the equation $3x^2 - 6x + 2 = 0$, giving the roots correct to two places of decimals.

(ii) If the expression $y^2 + 4y - a$ is equal to -1 when $y = 0$, find its value when $y = -\frac{1}{2}$.

4. (i) Find the value of x and the value of y which satisfy the equations

$$2x - y = \frac{3}{7}(2x + y) = 6x - 7y + 4.$$

(ii) Given that $\frac{5a + 4b}{a + 3b} = \frac{3}{2}$ find the value of $\frac{a}{b}$.

5. (i) Without using tables evaluate

$$(a)\ \left(\frac{25}{9}\right)^{\frac{3}{2}} \qquad (b)\ \left(3\tfrac{3}{8}\right)^{-\frac{4}{3}}.$$

(ii) (a) Express, as a power of x, $x^a \times x^b \div x^{a-b}$.

(b) Express, without brackets and with positive indices, $xy(x^2y^{-1} + x^{-2}y)$.

(iii) Two motor cars started together to race over a distance of a miles. One car had an average speed of x m.p.h. and crossed the finishing line b minutes ahead of the other. Find an expression for the time in hours taken by the slower car.

6. £5,640 is divided between three people A, B and C in such a way that A receives £1,000 more than B, and B receives $\frac{3}{4}$ of the amount C receives. Find the amount each receives.

Section B

[*Answer any* THREE *questions in this section*.]

7. (i) Use logarithms to find the value of $\frac{0\cdot 3818 \times 216\cdot 8}{0\cdot 0777 \times 5\cdot 691}$

(ii) If $V^2 = \frac{a^3 + b^3}{36\pi}$ find the value of V when $a = 2{\cdot}671$, $b = 0{\cdot}9343$ and $\pi = 3{\cdot}142$.

8. A groundsman has 176 ft. of fencing to make a rectangular enclosure. If he makes the width x ft., what is the length? Show that the area enclosed is $(88x - x^2)$ sq. ft.

Draw a graph of this function for values of x from 20 to 80, taking 1 in. to represent 10 ft. on one axis and 200 sq. ft. on the other, and from it find:–

(*a*) the greatest possible area which can be enclosed and

(*b*) the length and width of the enclosure when the area is greatest.

9. (i) Find the value of a and of b if $x + 2$ and $x - 1$ are factors of $3x^3 + ax^2 - bx + 4$. What is the remaining factor of the expression?

(ii) Solve the simultaneous equations

$$x^2 - xy + y^2 = 19$$
$$x + 2y = 4.$$

10. (i) (*a*) The first and last terms of an arithmetic progression are 213 and 133, and the sum of the progression is 2,941. Find the number of terms in the progression.

(*b*) The sum of the first twenty terms of a geometrical progression is 13 and the common ratio is $\frac{1}{3}$. Find the first term and the seventh term.

11. A and B are two towns 44 miles apart. A cyclist and a motorist travel from A to B. The motorist leaves A 1 hr. 36 min. later than the cyclist but they reach B at exactly the same time. If the average speed of the motorist is 18 m.p.h. greater than that of the cyclist, find the average speed of each.

UNIVERSITY OF LONDON

GENERAL CERTIFICATE OF EDUCATION

Ordinary Level

SUMMER, 1957

PURE MATHEMATICS

(c) GEOMETRY

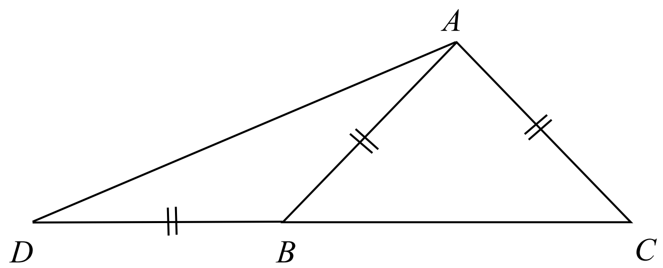
TUESDAY, *June* 25. – *Morning*, 9.30 *to* 12

All necessary working must be shown

SECTION A

[*Answer* ALL *questions in this section*.]

1. (i) In the figure $AB = CD = BD$ and the angle $ABC = 40°$.



Calculate the number of degrees in the angle CAD.

(ii) $ABCD$ is a trapezium with AB parallel to DC. If the angle $B = 50°$ and the angle $D = 120°$ find the number of degrees in each of the angles A and C.

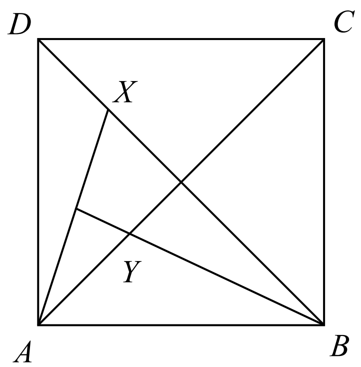
(iii) Three interior angles of a pentagon are 90°, 120° and 130°. If the remaining angles are equal find their common value.

2. $PQRS$ is a trapezium in which PQ is parallel to SR; $PQ = 10$ in., $PS = 3$ in., and $SR = 4$ in. If PS and QR are produced to meet in X find

(i) the length of XS,

(ii) the ratio of the area of triangle XSR to the area of trapezium $PQRS$.

3. In the figure, $ABCD$ is a square, X is a point in the diagonal BD and the perpendicular from B to AX meets AC in Y.



Prove that triangles DXA and AYB are congruent.

4. $ABCD$ is a square of side 6 in. and P is a point in CD such that $CP = 3{\cdot}5$ in. Calculate

(i) the area of triangle ABP,

(ii) the length of AP,

(iii) the length of the perpendicular from B to AP.

5. Construct the quadrilateral $ABCD$ given that $AB - AD = 8$ cm., $BC = CD = 6$ cm., and the angle $ABC = 75°$.

Construct the point K on AB produced such that triangle AKD is equal in area to the quadrilateral $ABCD$. Measure AK.

6. (i) E is a point outside a circle. EBA, EDC are two straight lines drawn to cut the circle in B, A and D, C respectively. If $AD = DE$ prove that $BC = BE$.

(ii) P, Q and R are points on the circumference of a circle, centre O. The angle $ORP = 20°$, the angle $ROQ = 80°$ and P, Q lie on the same side of the diameter through R. Calculate the number of degrees in the angle PQO.

SECTION B

[*Answer any* THREE *questions in this section*.]

7. P is any point on the median AD of a triangle ABC. BP is produced to meet AC in L and CP is produced to meet AB in M. AD is produced to X so that $DX = PD$.

Prove that (i) $\dfrac{AM}{AB} = \dfrac{AP}{AX}$ and

(ii) ML is parallel to BC.

8. Prove that tangents to a circle from an external point are equal, and equally inclined to the line joining the point to the centre of a circle.

Two parallel tangents to a circle, centre O, are cut by a third tangent in X and Y. Prove that XOY is a right angle.

9. Prove that if two circles touch, the point of contact lies on the line joining their centres.

P is a point distant 10 cm. from the centre of a circle of radius 5 cm. Construct two circles each of radius 3·5 cm. to pass through P and to touch the original circle. If these two circles intersect again in Q measure the length of the common chord PQ. Show all construction lines.

10. $ABCD$ is a quadrilateral in which AB is parallel to DC. By drawing perpendiculars from C and D to AB prove that

$$AC^2 + BD^2 = AD^2 + BC^2 + 2AB.CD.$$

11. Prove that the line which bisects the vertical angle of a triangle internally divides the base internally in the ratio of the sides containing the angle.

The tangent at a point T of a circle, centre O, meets a radius OA produced in X. TN is the perpendicular from T to OA. Prove that $NA : AX = NT : TX$.

UNIVERSITY OF LONDON

GENERAL CERTIFICATE OF EDUCATION

Ordinary Level

SUMMER, 1957

PURE MATHEMATICS

(g) HISTORY OF MATHEMATICS

[*TWO AND A HALF HOURS are allowed.*]

[*Candidates should write an essay on* ONE *of the topics in question* 1 *and answer* FOUR *questions from the rest of the paper. It is suggested that about* $\frac{3}{4}$ *hr. should be spent on question* 1.]

1. Write an essay on *one* of the following topics:–

(*a*) The earliest records of mathematics.

(*b*) Pythagoras an numbers.

(*c*) Newton and Leibniz.

(*d*) The influence of astronomy on the early history of mathematics.

(*e*) Incommensurable numbers.

2. Why can Archimedes be regarded as the originator of the integral calculus? Who could be regarded as the originator of the differential calculus?

3. School mathematics is usually divided into the sections (i) algebra, (ii) arithmetic, (iii) geometry, (iv) trigonometry, (v) calculus. Which of these sections flourished at the following periods:– (*a*) before the birth of Christ, (*b*) the first 1000 years A.D., (*c*) between 1000 A.D. and 1500 A.D., (*d*) between 1500 A.D. and 1750 A.D.? Briefly describe the development of mathematics which occurred during this last period.

4. Give examples in which the practical needs of society caused the development of a branch of mathematics.

5. Write a short account of the discovery of logarithms.

6. The mathematician Kronecker once said, “God made the integers, all the rest is the work of man”. Explain this.

7. Give an account of the most common signs used in mathematics.

8. Name *three* English mathematicians, stating approximately the times in which they lived and their chief contribution to mathematics.

9. What are the regular polyhedra? Who were the mathematicians responsible for the study of them, and what did each contribute?

10. Write a brief description of *two* instruments of calculation, explaining how they are used.