First stage of Israeli students competition, 2011.

14/1/2011

Duration: 4 hours

1. Find all possible values of $\lim_{x\to\infty} x^{(\ln x)^{\lambda}}$ for real λ.

2. Is it possible to draw a pentagon with integer coordinates of vertices and equal sides?

3. Compute $1+\frac{1}{2}-\frac{2}{3}+\frac{1}{4}+\frac{1}{5}-\frac{2}{6}+\frac{1}{7}+\frac{1}{8}-\frac{2}{9}+\frac{1}{10}+\frac{1}{11}-\frac{2}{12}+\ldots$.

4. Michal and Ohad play a game in which Michal marks points and arcs in the plane, and Ohad assigns colors to the points. Michal makes the first move. In each of her moves, Michal marks one point, and she can also join it by arcs to some of the existing points, provided that the arcs do not intersect (except possibly in endpoints). Ohad, in turn, paints the last marked point in some color, which must be different than colors of endpoints connected to this point by an arc. Michal wins, if Ohad will use more than 5771 colors. Does Michal have a winning strategy?

5. An infinite sequence of positive real numbers satisfies:

$$\det\begin{pmatrix} 1 & a_i & 0 & a_i^2 \\ 2 & \sqrt{3}\cdot a_{i+1} & a_{i+1} & 2a_{i+1}^2 \\ 2 & a_{i+2} & \sqrt{3}\cdot a_{i+2} & 2a_{i+2}^2 \\ 1 & 0 & a_{i+3} & a_{i+3}^2 \end{pmatrix} = 0$$

Prove that it is periodic.

Good luck!