The De Morgan Journal 2 no. 4 (2012), 1–4 ISSN 2049-6559

CAN MATHEMATICIANS HELP?

DAVID G. WELLS

Abstract

Professional mathematicians have not made the contributions to the teaching of mathematics in schools that might have been expected, in part, at least, because of their failure to appreciate the processes of conceptualisation and reconceptualisation that lie behind good maths teaching and lead young children from naïve concepts, objectionable perhaps to the professional, in time to more sophisticated and professionally acceptable interpretations. Illustrated by the idea that 'Multiplication is repeated addition.'

Common sense suggests that professional mathematicians ought to have a great contribution to make to the teaching of mathematics in primary and secondary schools, not least because they are, after all, the experts on the nature of mathematics (but see also the arguments in Wells [**3**]) and on how mathematics is done.

Yet when professionals have been prominent on the scene, for example during the New Maths era, the results have usually been disastrous. Why might this be? We can illustrate the difficulties and potential pitfalls with an example: let's take the important idea that "multiplication is repeated addition".

Strictly speaking, this is false. Professionals do not define multiplication as repeated addition—though to be fair, given the axioms of an integral domain or a field, you can actually only perform multiplication—once you have a system with which to count such as the Indo-Arabic system of numerals which is no part of the axiomatic structure—by the use of the distributive law, which leads directly to the simplest method of calculating, say, 6×4, as either $6+6+6+6$ or $4+4+4+4+4+4$.

More complicated products which involve fractions inevitably involve, from the same professional perspective, the properties of the multiplicative inverse, but the element of repeated addition is still strongly present. The simplest way for a 10-year-old, for example, to multiply $4\frac{1}{2}$ by $2\frac{1}{4}$ is to argue that it is equal to 4 times $2\frac{1}{4}$ plus one half of $2\frac{1}{4}$ which makes 9, by in effect repeated addition, plus $1\frac{1}{8}$. The

2000 *Mathematics Subject Classification* 97C50 (primary), 97D20 (secondary).
Email: *davidggwells≫at≪yahoo.co.uk*

latter partial sum involves the idea that $\frac{1}{2}$ of $\frac{1}{4}$ is equal to $\frac{1}{8}$, because, basically $2\times4 = 4+4 = 8$, once again bringing in (minimally) repeated addition.

Multiplication of even more complicated numbers, including the simple calculations involving π which occur in finding the perimeter and area of the circle, take the teacher and pupil even further away from the simple and naive idea that multiplication "is the same as" repeated addition, yet the basic idea never entirely disappears, not least because it is after all only a simple implication, when at least one of the multipliers is an integer, of the distributive law.

There is a crucial tension here between the highly abstract perspective which comes naturally to the professional mathematician and the extremely concrete perspective that is natural to the teacher who knows that pupils are expected to start to understand potentially very complicated ideas—such as multiplication—when they are very young and long before their ability to understand abstract ideas is well-developed, if it ever will be. (The vast majority of pupils never get anywhere near the level of the professional, in mathematics as in any other subject.)

The natural career of a concept such as multiplication-as-repeated-addition [MARA], for the teacher, is from very simple situations in which the MARA interpretation is entirely justified—frankly, it is simply following the distributive law for integers—into situations in which the idea of MARA requires modification as additional features enter the scene (such as multiplicative inverses) or even greater transformations as one of the original features of multiplication is lost (multiplication of matrices).

During this extended career, which for many pupils will go from infant school up to GCSE, and for some pupils to A-level, followed by degree courses—during which multiplication will be understood from far more abstract perspectives—the concept is repeatedly built on, by adding new aspects or new features, and reconceptualised, as previous understandings are modified or transformed.

This may sound like a very complicated process. It is, though it is one that good teachers are entirely accustomed to and take the granted—and their pupils should also find it familiar. They should not be confused, and if properly taught will not be, if the teacher says to them vis-a-vis some topic or in answer to some question,

"I'll explain the idea in the simplest possible way. You will understand it better next year."

After all, the same phenomenon, of a concept starting with a simple meaning but then developing more and more sophisticated meanings or interpretations as the pupil's studies progress, is found in every subject. The French Revolution, to a 10-year-old, ought to have become a very different concept by the time the same pupil reaches the age of 16, or 18 or 20.

Unfortunately, professionals do not always appreciate this subtlety. My choice of MARA as an example was prompted by one of Keith Devlin's *Devlin's Angle* columns [**1**], on the Mathematical Association of America Online website, dated June 2008, which displayed just this weakness. Devlin not only started by claiming that, "multiplication simply is not repeated addition" but went on to claim that "Telling students falsehoods on the assumption that they can be corrected later is rarely a good idea", demonstrating his failure to grasp how mathematics teaching works.

Not surprisingly, his claim stirred up a great deal of argument, leading him—and his critics—to return to the theme again—and again—and again. My own eventual response was the article *What is multiplication?* in Mathematics in School, May 2012 [**2**].

Keith Devlin's problems arose in the first place because he had no idea at all of the manner in which a concept can be presented simply, initially, and then developed, built upon, expanded, reinterpreted and reconceptualised, as pupils get older, as their capacity for abstract thought increases, and as the mathematics they are studying becomes itself more complex and demanding.

We are today once again entering an era when many professionals may feel that it is their duty, as well as their right, to make their views clear—not least because the present government wants universities to help draw up new A-level examinations, and in some as yet unspecified manner to assist them in making such new examinations more "rigourous".

At this point teachers of mathematics are likely to feel their spirits sinking. After all, there is little doubt that Keith Devlin honestly believed he was being rigourous in objecting to the MARA concept.

I am sure that many teachers will welcome the contributions of professional mathematicians, provided that they can see evidence that the professionals do appreciate what the task of the teacher actually is and what (the best) teachers are trying to do. I hope that these comments will have contributed to that understanding.

References

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2. D. G. Wells, What is Multiplication?, *Mathematics in School*, 41 no. 3 (2012), 37–39.
3. D. G. Wells, *Games and Mathematics: Subtle Connections*, Cambridge University Press, to be published 31 October 2012.

About the author

The author has the rare distinction of going up to Cambridge on a mathematics scholarship but then failing his degree, though he did become British u-21 chess champion in 1960. He subsequently became a teacher in primary and secondary schools, and then an author of books such *What's the Point? Motivation and the Mathematics Crisis*, and a contributor to several journals of mathematics education. His latest book is *Games and Mathematics: Subtle Connections*, to be published by Cambridge University Press on 31 October 2012.

Email: *davidggwells* ≫*at*≪*yahoo.co.uk*