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NOMINATION AND DEFINITION

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Nomination is a principal ingredient of education and transfer of knowledge. Nomination differs from definition. The latter implies the description of something new with the already available notions. Nomination is the calling of something, which is the starting point of any definition. Of course, the frontiers between nomination and definition are misty and indefinite rather than rigid and clear-cut.

Any child, meeting an unknown animal in the Zoo, will ask: "What is this?" The answer: "An opossum" will suit him usually. The sounds of the word "opossum" are quite familiar to the young person. The name is identified with a new image and this knowledge suffices. The definition of opossum for an adult will contain something of the sort: "*Didelphimorphia* is the order of common opossums of the Western Hemisphere." There are no grounds to assume that the approach of a child is less reasonable that the attitude of an adult.

The process of acquaintance or introduction of strangers reminds us of a meeting of a child and opossum. The phase "Meet Joe Blake" carried no information about Brad Pitt prior to the famous remake by Martin Brest. The name of a person we meet for the first time is important but little informative, enabling us to determine just a collection of namesakes. Only the number of a social security card in the USA on the requisites of a passport in Russia may serve as a unique identification of a taxpayer from the point of view of an agent of the Internal Revenue Service. However, it is highly improbable to find a pedant so meticulous that he will recite the social security number of an old crony he introduced.

Science is impossible without concepts to be clarified and explicated in definitions. The functioning of science is in a sense the development of concepts. There are no grounds to believe that science uses the laws of nomination and definition which are completely incomprehensible in the simplest example of the meeting of a child and opossum.

Definition is rational, whereas nomination is universal. It is not by

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chance that nomination plays a key role in the versatile manifestations of mysticism, occultism, and religion. The lexicon of science consists of concepts. The evolution of concepts is a historical witness revealing the features of bygone times. The traces of any epoch are reflected in its most abstract concepts. Neglecting the historical background makes it impossible to understand correctly not only the generally accepted concepts but also the brand-new terms like nanotechnology or quantum logic.

Mental continuity is a priceless gift enabling us to preserve the experience of our ancestors. The first transfinite act of the mankind is the birth of the idea of the collection of all natural numbers. The idea of actual infinity remains a challenge for the intellectual efforts of the scientists of all eras and states from *Metaphisics* by Aristotle and *Psammith* by Archimedes. The monads of Leibniz alongside the fluxions and fluents of Newton are products of the heroic epoch of telescope and microscope. The von Neumann universe of the middle of the twentieth century implements the Pythagorean thesis that "all is number." Measuring infinity by number is the crux of the revealing research of the genius Cantor. So long was the crooked way of the mysteries and nominata of reason from Paleolith to this day.

We find the origins of the modern science in Ancient Greece. The books by Euclid are the most important sources of scientific traditions. From the antic times, geometry deals with the quantitative and qualitative properties of spacial forms and relations. The criteria for equality of triangles provide instances of qualitative geometric knowledge. Finding lengths, areas, and volumes exemplifies quantitative research. The incommensurability of the side and diagonal of a cube became an outstanding discovery of Euclidean geometry. It was the first time when science has confronted the problem of counting the continuum.

When our ancestors had demonstrated the absence of any common measure of the side and diagonal of a cube, they understood that rational numbers are scarce for practical purposes. It is worth recalling that the set of rational numbers is equipollent with the collection of natural numbers. This means that all rational numbers comprise a countable set, thus serving as an instance of the cardinal number that we use to express the size of the imaginary collection of all entries of the natural series. The discovery that the side and diagonal of a cube are incommensurable is the height of mathematics as awesome and ethereal as the independence of the fifth postulate, the axiom of choice, and the continuum hypothesis.

The definitions of Euclid's *Elements*, the greatest scientific treatise in the history of the mankind, reflect the geometric vision of the world of his epoch. Geometry is part of the culture of the ancient world which was invented to meet various human needs. Its mystic, explorative, and economical sources coexisted in the common cultural environment of the man of the pre-Bible times. The strongest quest of geometry stemmed from the cadastral surveying aimed at regular taxation. The famous *harpedonaptae* of Egypt were tax agents who used ropes for measuring the tracts of land. The tricks and techniques of *harpedonaptae* were used in construction. Pyramids were erected long before the abstract definition of the geometrical form of a pyramid. It is impossible to ignore the fact that the rope stretching taut between two stakes is a mental icon of the concept of a straight line segment which is the continuum of the modern mathematics. The problem of the continuum, the greatest puzzle for the first mathematical minds of the twentieth century is a shadow of the practical task of commensuration of diverse fragments of lines.

Bewildering is the history of the abstract geometric concepts of point, monad, figure, and solid which came from the remote ages. We are rarely aware of the fact the secondary school arithmetic and geometry are the finest gems of the intellectual legacy of our forefathers. There is no literate who fails to recognize a triangle. However, just a few know an appropriate formal definition. This is not by chance at all, since the definition of triangle is absent in the Elements. Euclid spoke about three-lateral figures, emphasizing that "a figure is that which is contained by any boundary or boundaries." Clearly, his definitions remind us of the technology of cadastral surveying of his times. It is worth observing that the institution of property is much older than the art and science of geometry. To measure a tract of land from outside is legitimate whereas trespassing the borders is forbidden. The ancient rope stretchers had similar restrictions for measuring the constructions like pyramids. Clearly, the surveyors of the Kheops pyramid would mum every single word about the interiors of this building.

In the modern parlance, we say that Euclid considered convex figures and solid bodies. The concept of convexity seems quite elementary today. Part of a plane or space is called convex provided that no straight line segment between any two points of this part lies within the object under consideration. If we drive three stakes in a tract of land and stretch a lasso whose loop surrounds the stakes, we will single out a triangle. The harpedonaptae did exactly the same, but the interior of the tract to be measured might be inaccessible to the surveyors

without permission of the owner. Nowadays we also measure property and levy taxies but any unauthorized attempt to stretch a rope within somebody's property is still a felony of trespassing on land. The definitions of Euclid are listed among the immortal witnesses of the ancient economical relations.

Mathematics is the fist science of the knowing man. Homo sapiens perceives the reality and himself in the external world by the physiological methods that enable him to discern items and distinguish their forms. The abstract forms and relations of the human mind are the starting points of man's scientific nomination and definition. Euclidean geometry is an exemplar of rational creativity for two and a half millennia. The modern science possesses many hundreds of new theories, while nominating and defining hundreds of thousands of the new objects and concepts unknown to Euclid. However, the method of scientific research remains practically the same. Euclid could learn the fundamentals of any appealing present-day scientific discipline with the same ease as the children of any race or nationality throughout the world master the basics of Euclidean geometry.

Perseverance of generations is a token of the immortality of science.

About the author

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