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NURTURING ABLE YOUNG MATHEMATICIANS

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ABSTRACT

We summarise the developments of the last 20 years—highlighting the key underlying assumptions, and indicating certain unfortunate consequences. We show how official policy has been based on

– persistent failure:
  (i) to develop and to implement a suitably challenging curriculum, and
  (ii) to provide ordinary teachers with good texts, suitable subject-specific professional development, and appropriate assessment targets;
– a misconception of the curriculum as a **one**-dimensional 'ladder' (with each topic *nominally* the same for everyone, with uniform expectations for all pupils at a given 'level'), up which pupils progress at their personal rate, and
– associated accountability measures that have unintended consequences.

We then outline the alternative conception of a **two**-dimensional "$*$-curriculum", in which each theme in the standard curriculum sequence is explored (and where necessary, assessed) to different *depths*, and where those who manage to dig deeper and to lay stronger foundations emerge naturally as the ones who are well-placed to subsequently progress further. In such a model, able pupils in Years 5 and 6 would **not** be pushed ahead to achieve a premature and superficial mastery of 'Level 6' material, but would spend time exploring harder problems at 'Level 4' and 'Level 5' (so-called 4* and 5* material). Similarly, able students in Years 10 and 11 would **not** be entered *early* for an accessible but superficial GCSE, but would instead be expected to master core GCSE material more deeply, so as to make the subsequent transition to A level in Year 12 straightforward.

The present UK administration and its predecessor have both made efforts to develop a more mathematically literate population, and in particular, to forge a system which routinely generates more young people who leave school willing and able to pursue mathematically demanding degree courses and occupations. Thus, on one level there would seem to be a clear recognition that failure on this score will result in continued under-performance in many fields essential to the economy and to society in the 21st century. Indeed, the speeches on this theme given by the Secretary of State over the last two years have often begun with wide-ranging and well-drafted comments that suggest a profound sensitivity to mathematics, to its history, and to its significance. The themes addressed in these speeches are important and are to be welcomed: to strengthen the National Curriculum; to involve Higher Education once more in the design and operation of A levels; to explore the possibility of specialist mathematics provision for some aged 16-18; etc.. However, effective practice and policy in such delicate areas must respect the insights of those with relevant experience and must demonstrate an acute sense of what is needed, and what could work, in ordinary schools. So professional inputs are needed at an early stage. Yet current policies and initiatives suggest that three key ingredients are consistently missing:

(i) There seems to be little prior discussion and analysis of the problem being addressed.

2000 *Mathematics Subject Classification* 00A05 (primary), 01A55 (secondary).

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(ii) There is no evidence that anyone has checked that the resulting proposals 'could fly' (i.e. that they take account of the realities 'on the ground').

(iii) There is no recognition of the need for carefully planned *pilot* projects to evaluate and to improve the basic design *before* large scale implementation.

One result is that potentially valuable and knowledgeable inputs from the mathematics and education communities are sought too late. This paper looks at some of the evidence from the last 20 years with particular reference to *nurturing potentially able students*. It outlines an approach, which has been tried and tested in different ways, and which has generated a surprisingly broad consensus among those with experience in this area. The approach avoids the mistake of imagining that 'ability' is God-given, and focuses on devising a curriculum, classroom materials and instruction, and an assessment structure which allows high achievers *to emerge naturally*, and in large numbers, without the need to label students prematurely as 'sheep' and 'goats'.

To make sense of what emerges as we proceed, the reader may find it helpful to focus on provision for those students (roughly 25–30% of each cohort) whose studies after the age of 16 would benefit from a stronger grounding in GCSE mathematics, and who should *ideally* reach this age with the mathematical competence to decide whether to continue with their mathematical studies and if not, to feel confident that they can use what they have learned in whatever subjects they choose to study. This 'ideal' requires that around 30% of the cohort should have a much deeper grasp of elementary mathematics prior to GCSE than is provided in the current system (a system which is dominated by 11–16 schools, whose horizons are limited to achieving Grade B/C, rather than preparing students for A level studies).

Some history

The remit of the recent National Curriculum Review group (for England) included the instruction to:

> "*develop a National Curriculum that acts as a benchmark for all schools and provides young people with the knowledge they need to move confidently and successfully through their education, taking into account the needs of different groups including the most able*".

The draft curriculum recently out for consultation fails to address the long-term needs of potentially able students sufficiently explicitly. Given that the prevailing rhetoric recognises that our economic future depends on high added-value, such neglect of a key national resource is striking.

However, given the widespread assumption that able pupils are somehow 'bound to survive' even if the system neglects their needs, this oversight is unlikely to be widely noticed. Though prevalent, the assumption that able pupils will survive, come what may, has been without foundation for at least 25 years. Our systemic failure on this score has been understood to some extent by those at the highest levels; yet their responses have alternated between (a) inaction, and (b) ill-conceived programmes that deliver headlines, but very little of any lasting value (as was the case with the *Gifted and Talented* strand within *Excellence in Cities* 1999–2006; the millions spent on *MoreMathsGrads* and the subsequent HE-STEM programme; and the recent 'access' schemes to universities).

In 1995 the three mathematical learned societies (LMS, IMA, RSS) produced a joint report *Tackling the mathematics problem*[†], which highlighted academic concerns regarding the quality of those then entering numerate degree courses in UK universities. The report's findings were initially dismissed by government. But in the months that followed other evidence which seemed to support its analysis began to filter through (e.g. as the results from TIMSS (1995)), and official attitudes began to change.

TIMSS (1995) showed that average performance in English schools was weak; but the results also showed that, in comparison with a broad range of other countries:

> only **7%** of English **Year 9** pupils performed at or above the top 10% level (roughly the 'most able'),

and

> only 20% scored at or above the top 25% level.

Germany achieved 6% and 25% respectively; and the USA 5% and 18%. *All other comparable countries performed significantly better than England*:

> France: 7% and 26%; Canada: 7% and 25%; Ireland: 9% and 27%; Netherlands: 10% and 30%; Australia: 11% and 29%; Hungary: 11% and 29%; Austria: 11% and 31%; Flemish Belgium: 17% and 41%; Czech Republic: 18% and 39%[†].

Such figures deriving from a single 1995 sample should of course be interpreted with care. However, the underlying message was reinforced by the results in the linked 1995 TIMSS study for Year 5 pupils, which showed that (*ibid* page 10):

> just 7% of English pupils scored in the 'top 10%'; and only 17% scored in the 'top 25%', with these figures placing England at the bottom of the list of comparable countries (ranging from Canada: 7%, 21%; USA: 9% and 26%; Ireland 10% and 28%; up to Netherlands 13% and 36%; and Czech Republic: 15% and 34%).

The subsequent study TIMSS (1999) was restricted to Year 9s. A number of stronger countries had dropped out after 1995, and new developing countries had joined in—making it easier to score in the 'top 10%', or 'top 25%'. But the results were scarcely more encouraging, with 7% of English Year 9s scoring among the top 10%, and 24% scoring among the top 25%.

> Among comparable countries only Italy fared worse, with 5% and 20% respectively. USA managed 9% and 28%; Czech Republic: 11% and 33%; Australia: 12% and 37%; Netherlands: 14% and 45%; Hungary: 16% and 41%; Flemish Belgium: 23% and 54% [‡].

This suggests that, compared with England, many other countries were managing to produce two or three times as many 14 year olds who could solve mildly interesting problems.

TIMSS (2003) showed a marked improvement in average scores for Year 5 pupils (presumably as a result of the increased focus and confidence accompanying the Numeracy Strategy, which had by then influenced primary school practices for 3–4 years). But the figures for Year 9 pupils remained discouraging—with just 5% scoring at the 'advanced

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[†] `http://202.38.126.65/oldmirrors/www.lms.ac.uk/policy/tackling_maths_prob.pdf`.

[†] `http://timssandpirls.bc.edu/timss1995i/TIMSSPDF/PSA_M_all.pdf` (page12).

[‡] `http://timssandpirls.bc.edu/timss1999i/pdf/T99i_Math_1.pdf` (page 43).

benchmark' (a refinement of the previous 'top 10%') and 26% scoring at the 'high benchmark' (a refinement of the previous 'top 25%'). Again many of the original countries did not take part, but the England figures placed us at the bottom of the pile of comparable countries, with

> USA: 7% and 29%; Australia: 7% and 29%; Flemish Belgium: 9% and 47%; Netherlands: 10% and 44%; Hungary: 11% and 41% †.

In TIMSS (2007) more of the original participating countries dropped out. The results tended to confirm the improvement for Year 5 pupils (to a level comparable with Russia), and began to show a degree of improvement for Year 9s—with 8% and 35% performing at the 'advanced' and 'high' benchmarks (slightly better than the USA, Australia and the Czech Republic, but worse than Hungary) ‡.

Such studies provide evidence; but this evidence needs to be checked against other measures. PISA is a quite different, if much criticised, international study, which surveys 15 year olds in numerous countries every 3 years–using tasks that set out to test 'mathematical literacy', rather than recognisable school mathematics. PISA reports success at six levels.

> In 2009 their top level, level 6, was achieved by an average of 3.1% of surveyed 15 year olds, by 8% of those in Switzerland, by 5.8% in Belgium, 4.9% in Finland, 4.6% in Germany, 4.5% in Australia, 4.4% in Canada and the Netherlands, but by only 1.7% of those in the UK.
>
> PISA level 5 was achieved by 12.7% of surveyed 15 year olds, by 24% in Switzerland, 21.6% in Finland, 20.4% in Belgium, 19.8% in the Netherlands, 18.3% in Canada, 17.8% in Germany, 16.4% in Australia, but by just 9.9% in the UK§.

More disturbingly, the Sutton Trust analysis of the PISA data explicitly suggests (para. 6.11, p.43; para 7.21, p.49; see also para 7.12, p.46) that almost none of those performing at the highest level come from state schools. The Sutton Trust report's conclusions and recommendations are often unfounded; but it is hard to argue with the data¶.

Despite the difficulty of interpreting such figures precisely, it would seem clear that it has long since ceased to be the case that anyone could claim that able young mathematicians in the UK were being nurtured as one might wish. This observation is reinforced by admission trends to our best universities (where the number of successful home-grown applicants has drifted downwards during the last 10–15 years), by success in national Olympiads (where the lists of prize winners are increasingly dominated by a handful of 'recent arrivals' in the UK—a phenomenon to be seen also in the successful school teams in the National Team Challenges for Years 8–9 and for Years 12–13). Similar trends are clear among major UK-based firms who employ high quality mathematics graduates (many of whom recruit 80–90% of their younger employees from other nationalities), by the performance of the UK team at the annual International Mathematical Olympiad, and by recent patterns of mathematical appointments to positions in UK universities (which have been increasingly dominated by overseas candidates). *There are therefore numerous reasons to review and revise both the approach and the assumptions of the last 30 years.*

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†`http://timssandpirls.bc.edu/PDF/t03_download/T03_M_Chap2.pdf` (page 64).

‡`http://timssandpirls.bc.edu/TIMSS2007/PDF/T07_M_IR_Chapter2.pdf` (page 71).

§`http://www.suttontrust.com/research/educating-the-highly-able/`.

¶`http://www.pisa.oecd.org/dataoecd/10/61/48852548.pdf` (page 131).

The incoming labour government in 1997 recognised the underlying problem; but, rather than discussing the issue with those who had local experience in this area (especially in subjects like mathematics that were most directly affected), they appointed a non-specialist to seek an inappropriate North American 'analysis' and 'solution'. The approach they adopted was based on a belief that (mathematical) ability is fixed so that all one had to do was identify it and wait. This approach assumed that such potential develops largely on its own without any serious subject-specific input from teachers, and without requiring hard work on the individual's part.

The *Gifted and Talented* strand of the *Excellence in Cities* programme offered money to schools

(a) provided each school made lists 'identifying' 5–10% of pupils deemed to be 'more able' and outlined general (short-term) plans to show how those so listed were being encouraged, and
(b) provided the participating Local Authority met certain easily-measurable targets (in particular, for early GCSE entry).

But nothing was done to develop an 'extension curriculum', or teaching materials, or effective assessment, or systematic subject-centred CPD: that is, there was no investment in the tools needed for sustained, long-term development of potentially able young people.

The unfortunate consequences were exacerbated by the serious misconception of the curriculum as a single 'ladder'—up which every pupil was expected to scramble at his/her own pace. This misconception encouraged the belief that all that was needed was to encourage able pupils to scramble faster than everyone else up the same inadequate curriculum ladder: hence the requirement for EiC-participating Local Authorities to meet targets for early entry to GCSE. The accountability measures employed actively encouraged *superficial* acquaintance with 'harder' topics, rather than the kind of deep, fluent, and robust mastery of the curriculum that is needed for long-term mathematical progress.

Faced with the outworkings of the policy adopted in 1997–8, the wider mathematics education community foresaw the dangers and responded with the report *Acceleration or enrichment?*[†] and two follow-up reports. These reports united almost everyone in the mathematics and mathematics education communities. But the money and influence remained with the *G&T* strand of *Excellence in Cities*—whose influence is still seen in such unrelenting lobbying as that to be found at http://www.gtvoice.org.uk/.

Towards an effective national policy

Every maths teacher knows that some students appear to find school mathematics easier, or more natural, than others. But every effective maths teacher also knows that good teaching and sustained hard work can ultimately lead less evidently talented students to rival, or even out-perform, those who may have found mathematics easy from an early age. This should warn us all against pre-judging eventual outcomes, and against leaving bright pupils to look after themselves: like everyone else, potentially able pupils need to be challenged and stretched. In this regard we note that most of the recent arrivals in the UK who increasingly dominate our prize lists, our university places, and many of the best jobs

[†]UK Mathematics Foundation (2000), *Acceleration or enrichment?* Reprinted in The De Morgan Journal 2 no. 2 (2012), 97–124.

would never think of attributing their success to any fixed talent. They come from cultures which focus on the virtues of *hard work*, and the need for application over many years[‡].

An observant teacher often notices apparent potential. But s/he is well aware that the potential s/he notices is human: it therefore varies over time, and responds differently to different challenges (for all sorts of social, developmental, and psychological reasons). Like everything else, the development of robust mathematical talent is a long-term process that depends on many variables—including sustained hard work and quality instruction. To imagine that one can 'identify' and predict the eventual outcome of this extended process in advance is like picking probable winners in the early stages of a marathon race.

National policy therefore has to be designed

- to systematically generate ever larger numbers of potentially able young mathematicians, and
- to provide them with a day-to-day experience of school mathematics that encourages more of them to reach age 18 wanting to pursue seriously numerate studies at university.

The need for an effective policy to this end was reinforced by the recent report from the House of Lords Select Committee on Science and Technology[†].

We need to focus the minds of pupils, teachers, and government on the kind of active work that is needed by the state, by teachers, and by large numbers of students, if we are to improve the eventual outcomes.

In devising such a policy one has to start from the recognition that the goal is not to enhance pupils' scores on simple tests along the way, but

> to nurture large numbers
> of healthy, competent, committed students
> from ordinary schools
> who at the age of 18, or 21, or 25 are able to, and want to continue studying mathematics.

In contrast, the international data suggest that successive governments have repeatedly adopted policies that have effectively increased 'the class divide' with many 11–16, and even 11–18, schools concentrating on achieving the required 'three levels of progress' (from 'Level 5' at Key Stage 2 to Grade B at GCSE), so failing to lay the kind of deep foundations and attitudes that nurture future success in mathematics. We have invested large sums in programmes (such as *MoreMathsGrads* and its successor HE-STEM) which have been contractually prevented from addressing the kind of issues which might improve matters long-term (e.g. the curriculum, textbooks, teacher training and support, and suitable assessment goals). In the same spirit, improved 'access' to HE for under-represented groups has avoided facing up to the cause of inadequate preparation, and has instead expected universities to compensate for years of poor preparation without requiring anyone to address the inappropriate teaching and assessment students receive in Years 7-13. The recent House of Lords report intimates that the root cause must be addressed if we are not to face a bleak future.

The $*$-curriculum mentioned in the abstract offers a simple framework for addressing the root cause of these failures. The underlying principles emerged during the 1980s and early

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[‡] `http://www.gladwell.com/outliers/index.html`.

[†] `http://www.publications.parliament.uk/pa/ld201213/ldselect/ldsctech/37/37.pdf`.

1990s as part of a wider movement that eventually led to setting up the UK Mathematics Trust in 1996[‡].

These principles were first elaborated in the report *Acceleration or enrichment?* (UK Mathematics Foundation, 2000), and are still relevant. Instead of pretending one can 'identify' able pupils, the ∗-curriculum embraces the principles underlying all good teaching—namely that one should:

– provide a sufficiently *rich diet* (without acceleration) *for as large a group as one can manage* (say 25–30% of each cohort), with appropriate accompanying materials, CPD, and assessment,

and then monitor outcomes and tweak inputs to ensure that

– increasingly large numbers of high achievers (who respond to this opportunity, and who do the necessary work) emerge at age 16 or 18, wanting to continue with the serious study of more mathematics.

This approach has much in common with that adopted by the startlingly effective *Further Maths Support Programme*.

The principles underlying the ∗-curriculum were further elaborated in the 2003 Mathematical Association publication *Making better use of mathematical talent*[†], and in the CIMT publication *Beyond the soup kitchen*[‡]. These extended drafts outlined an approach which actively avoids accelerating able pupils on to easy material designed for older students. Instead the documents explained that what is needed is that potentially able pupils be routinely expected to master essentially the same material as their peers but more robustly and more deeply, and with a greater emphasis on making connections (so that they are regularly faced with problems whose solution requires links to be made, and for simple ideas from different spheres to be combined). That is, if students are ultimately to go further, they need to achieve a more profound mastery of core material before moving on. The spirit of the ∗-curriculum can be inferred to some extent from the attempt to present elementary mathematics and its inter-connections in the draft curriculum[§]. It is also exemplified in greater detail in the exercise sets, and in the introductory essay for teachers to be found in the series Extension mathematics (Oxford, 2007)[¶].

We give a brief example here. Such an approach would seek to teach, and to achieve routine fluency in using,

(a) how to find the equation of a straight line through two given points, the equation of the line through one given point with a given gradient, the proof of the relationship between the gradients of two perpendicular lines, and
(b) how to solve quadratics by factorising and by completing the square.

But it would also expect students *to apply* these techniques, for example,

(c) to find the centre of a circle (by completing the square);
(d) to use this to find the equation of the tangent to a given circle at a given point,

and would routinely expect the (modest, but important) flexibility that is required *to combine* these ideas in order

[‡]http://www.mathcomp.leeds.ac.uk/.

[†]http://202.38.126.65/oldmirrors/www.m-a.org.uk/docs/library/2462.pdf.

[‡]http://www.cimt.plymouth.ac.uk/journal/gardiner.pdf.

[§]http://education.lms.ac.uk/wp-content/uploads/2012/02/KS_1-4_DMJ.pdf.

[¶]http://ukcatalogue.oup.com/product/9780199151509000.do#.UCEPY02PVbE.

(e) to find the points where a given line meets a given circle,
to find the tangents to the circle at these two points, and
to find the coordinates of the point where these two tangents meet.

In the late 1980s and early 1990s, a significant number of mathematics teachers in state schools followed this approach, with marked positive effects. For example, the typical origins of the UK International Mathematical Olympiad team (of six students each year) reversed from 'five private and one state school student' to 'five state school and one private'; the mix also changed from 'one female in 24 years' to one or two females every year.

Increased use of targets and accountability since 2000 has made life more difficult for most state schools—and similar pressures have also affected many private schools. And though much of the supporting voluntary effort continues (such as the work of the UKMT related to the national Challenges, Olympiads, summer schools, and correspondence programmes), negative pressures on schools limit its impact. Nevertheless some schools continue to apply the principles implicit in the three bullet points above, despite pressures to achieve more easily measurable short-term outcomes. Such a $*$-curriculum is entirely consistent with recent Ofsted reports[†]. The most recent of these reports explicitly addresses in some detail quality provision for the most able.

Conclusion

We need to return to the basic principle that any effective national strategy for nurturing talent

- must begin by rejecting the idea that 'talent' is fixed and easily identified (and can then be 'treated')
- must learn from experience that 'acceleration' is rarely supportive of a student's long-term mathematical development, so should form no part of any core national policy (even if in individual cases it may occasionally be deemed to be the best option).

The DfE recently (November 2011) produced a report[‡] which should have settled the debate about acceleration once and for all. Yet even this report hesitates to draw the obvious conclusion so perhaps it needs to be stated explicitly. In general, *acceleration encourages superficiality*. It eventually leads many able students both to underperform relative to their potential and to lose their appetite for mathematics; and it increases insecurity among those students who need more time for ideas and methods 'bed in', but who if given that opportunity, might eventually blossom into highly competent mathematicians. Teachers and schools who understand elementary mathematics recognise that it is better for everyone if they concentrate on mastering Key Stage 4 (age 14–16) material in greater depth, gaining in confidence and competence by going further and deeper than is strictly needed for GCSE (in order to prepare for transition to subsequent study)—and then taking GCSE 'in passing' at the *end* of Year 11 and aiming to score 100%. Schools who take this line

[†] `http://www.ofsted.gov.uk/resources/mathematics-understanding-score` (2008), `http://www.ofsted.gov.uk/resources/good-practice-primary-mathematics-evidence-20-successful-schools` (2011), and `http://www.ofsted.gov.uk/resources/mathematics-made-measure` (2012).

[‡] `https://www.education.gov.uk/publications/eOrderingDownload/Early-entries-GCSEs.pdf`.

not only achieve better GCSE results, but end up with a much larger uptake of A level Mathematics and Further Mathematics.

Thus, before seeking to devise policies, we must

- recognise that useful talent is what emerges at the end of the teaching process
- devise a rich curriculum, appropriate teaching and suitable assessment
- target a student group that is sufficiently large as to include any who might benefit from what is on offer, and
- then be patient, and see which students take the bait and absorb the key lesson that *the rest is up to them and their own hard work*.

This approach highlights the desirability that the curriculum at any given stage should have associated objectives and extension material for those who can master core material more deeply, so that they are routinely expected to master topics in greater *depth*. For example, at Key Stage 2 (ages 7–11), such pupils should not be expected to achieve a flimsy grasp of secondary school mathematics, but need a regular diet of more demanding problems which *require no more than 'level 4' and 'level 5' content*. The experience of tackling harder problems that require one to combine familiar methods in order to solve elementary but unfamiliar multi-step problems is educationally far more important than to move on prematurely to poorly comprehended one-step routines from some 'higher level'.

The National Curriculum provides an outline *Programme of Study* for the whole cohort aged 5–16. Minimal provision in line with the specified *Programme of Study* (PoS) is sufficient *neither* to nurture mathematical ability, *nor* as a preparation for A level studies. So we need a "$*$-interpretation" of the listed content in the official PoS (with a few additional themes) to demonstrate the greater *depth* that is needed by those who might subsequently wish to proceed to further studies. In current circumstances this $*$-material must be assessed (if only to give teachers a clear idea of what is intended).

- Instead of including a 'level 6' test paper as part of the Key Stage 2 tests (at age 10–11), what is needed is a "$*$-paper" testing 'level 4' and 'level 5' material more deeply (or whatever may turn out to be the equivalent approach in the context of the new curriculum).
- At secondary school (Key Stages 3–4, age 11–16), schemes of work should routinely include an integrated $*$-*curriculum*, and assessment at age 16 should routinely be structured to include a harder paper—which should then serve as a prerequisite for A level Mathematics. (This could be done either (i) by restructuring the *Linked Pair* so that one GCSE is designed for almost the whole cohort, while the other "$*$-GCSE" is more demanding, or (ii) by adding a "$*$-paper" to each Mathematics GCSE.) Preparation for and access to such assessment should be statutorily available in all schools.

Details need to be worked out in the light of careful analysis. But any policies need to take account of the following principles, which we hope have been sufficiently justified in the preceding remarks:

- The mathematical potential of a given pupil is not fixed, but develops over time in response to the regular diet provided, and to sustained hard work.
- Quality *everyday* classroom experience is fundamental to good development, (though given such a rich regular diet, young people may also benefit from extracurricular provision from time to time).
- Any approach which depends on identifying and labelling a fixed group of young people as being particularly 'mathematically able' is ill-conceived and counter-productive.
- Whatever effective general policy is adopted, there is likely to be a small number of

very exceptional young people who may require bespoke provision. But they are so few, and their needs are so varied, that they are not relevant to the formulation of general policy affecting the 99.9%.

– In the absence of more effective policies, it is tempting (and cheap) to simply "accelerate" able pupils onto *standard* work designed for *older* students. Such acceleration in mathematics leads at first to easy success, but without developing the depth and tenacity which is needed for long-term progression; it also encourages shallow mastery which eventually breeds insecurity. Routine enrichment and extension work *with the minimum of acceleration* is far more effective in generating students' interest in and commitment to mathematics.
– Devising, structuring, and using a suitable programme of extension and enrichment material is a serious professional challenge which cannot be simply left to 'the market', or to schools or individual teachers (in part because the pressures placed on ITT, on schools, and on teachers in recent years have failed to generate the necessary expertise): teachers will need access to a suitable curriculum and assessment structure, and to quality materials—with opportunities to develop the necessary subject knowledge and subject pedagogical knowledge in order to use them effectively with their classes
– This issue is too important to be subject to politically-inspired change every few years.

The last two triennial Ofsted reports (*Understanding the score* (2008) and *Made to measure* (2012)) portray starkly the existing shortcomings in basic mathematics teaching—especially at Key Stage 3. A serious programme of specialist CPD is clearly needed! However, initiating such a programme requires a convenient catalyst: the advent of a $*$-curriculum and associated assessment, and the fresh professional demands this will make, would offer a natural opportunity to initiate such a programme.

Whatever is done to improve day-to-day classroom instruction, it is helpful to have a framework within which ordinary *classroom* and *additional* provision fit relatively naturally. Some systems (such as Hong Kong and Singapore) routinely distinguish between provision

(i) in school, in class;
(ii) in school, out of class;
(iii) out of school.

The first of these is by far the most important—and has been the central focus of this paper. However, we also need to encourage extra-curricular mathematical experiences which broaden horizons and help to establish a sense of belonging to a wider community of young mathematicians. Examples of the second category include maths clubs (whether focusing on stretching able students, or as a drop-in opportunity for those who are interested, but less committed); inter-house maths competitions; a sequence of sessions providing some practice for a larger group from which a representative school team may be chosen for the 'national team challenges'; visiting speakers from outside, who may work with students selected from different classes and from different year groups; and preparation for Olympiads. The third category ranges from local events organised by universities, or by learned societies, or by independent groups (such as MMP, or UKMT, or '*MathsInspiration*'), to residential Summer Schools. Some simple way of encouraging schools to develop such wider participation (such as an extension of the grants for schemes listed in the STEM Directory) is needed.