The De Morgan Gazette 9 no. 4 (2017), 17–25 ISSN 2053–1451

WHAT CAN SPECIALIST MATHEMATICS SCHOOLS GIVE TO STUDENTS THAT MAINSTREAM SCHOOLS CANNOT?

ALEXANDRE BOROVIK

Abstract

This paper contains
- some comments on the controversy around specialist mathematics schools,
- an attempt to clarify the nature of these schools and explain what makes them special, and
- a sketch of a realistic solution:

specialist mathematics classes run by universities in mainstream schools.

1. *Problem: controversy around specialist mathematics schools*

The recent Green Paper[†] sets the aim of

> *"expanding the number of specialist maths schools across the country"*. (p. 16)

In mass media, the reaction to this proposal was mostly negative, with statements along the lines that

> '*all students should receive excellent maths teaching – not just those in specialist maths schools*'

and claims that

> *specialist maths provision will only increase the divide between rich and poor standards in maths teaching.*

I can see only one plausible mechanism for the negative effect: specialist schools set along the blueprints of the existing Exeter and King's College London mathematics schools will attract the best mathematics teachers, luring them away from lesser schools.

A possible solution to this problem is relatively simple.

Why not go for a smaller number, smaller, but higher academic level specialist schools?

In these schools, mathematics should be taught by university teachers; this is why specialist mathematics schools have to be run by universities. This simple principle also sets a generic criterion of selection of pupils: they should be expected to benefit from being taught by a university lecturer. Such children do exist; in a rough assessment (broadly accepted by many of my colleagues who work with mathematically

---

2010 *Mathematics Subject Classification* 97A40, 97B10, 97C30, 97C60.

[†]HM Government, *Building Our Industrial Strategy.* Green Paper, January 2017. `http://bit.ly/2kh3roa`.

able children), there is one or two in a year in an average British school.‡ In an area with 20 or 30 secondary schools, some of these children can make a class taught at a much more advanced level than it is possible in a mainstream school. With this approach,

- no teacher will be taken away from mainstream schools;
- recruiting one or two students from a mainstream school will have limited impact on the school they are leaving;
- a specialist school would be able to provide not just "better" mathematics education, but a completely different kind of education – more on that later.

2. *Solution: Specialist Mathematics Classes*

Specialist mathematics schools form a continuous spectrum – ranging from ordinary schools with standard syllabus, through good schools with good maths teachers, to schools like Louis-le-Grand† in Paris and Fazekas‡ in Budapest. My comments apply mostly to the latter end of the spectrum.

The Government's plans "to open a specialist maths school in every UK city" – as reported in pre-announcements§ – are flawed in two ways:

- "in every city" is unfeasible – there are 57 cities in England and Wales;
- "school" is unfeasible as well.

2.1. *Why go the whole hog?* Instead, the Department for Education could create the necessary regulatory and financial framework for a much cheaper and simpler option:

- A University, a Local School, and the Department for Education sign a three-party contract setting up just two specialist mathematics classes, for example one for each of two years of 6th Form.
- Setting the mathematics curriculum, teaching, and examining mathematics in specialist classes is done by the University. This could apply to physics and computer science, too.
- Selection of students is also controlled by the university.
- DfE provides some funding – first of all, to compensate the University for staff time, for costs incurred in the selection procedure, etc., and to compensate the School for all the trouble, administrative costs involved, etc.
- Preferably, students should start specialist classes at age 14 – see my developmental argument in the next section, it applies primarily to young teens. But

‡A. V . Borovik and A. D. Gardiner, *Mathematical abilities and mathematical skills*, The De Morgan Journal 2 no. 2 (2012) 75–86. ISSN 2049–6559. `http://bit.ly/2jTYy4r`.

†A good description of Lycée Louis-le-Grand can be found in M. Lemme, *Utter elitism: French mathematics and the system of classes prépas*, The De Morgan Journal 2 no. 2 (2012) 5–22. ISSN 2049–6559. `http://bit.ly/2jJDYRs`.

‡P. Juhász, *Hungary: Search for mathematical talent*, The De Morgan Journal 2 no. 2 (2012) 47–52. ISSN 2049–6559. `http://bit.ly/2iW8mLg`.

§"*Theresa May will announce a drive to create a specialist maths school in every British city on Monday to ensure the country "stands tall in the world" after Brexit.*" The Telegraph, 21 January 2017, `http://bit.ly/2juf72o`.

classes for ages 16–18, which are easier to organise and run, could be useful, too.

Running specialist mathematics classes within a mainstream state school or an academy will help to avoid the stigma of social division so obvious in the grammar schools project. Helping mathematically inclined children to recognise their potential does not necessarily mean supporting the privileged. Classes rather than schools also make the endeavour affordable to smaller universities; I know at least one where my colleagues would be happy to run a couple of specialist classes in a local school – but definitely not a whole school.

After I produced the first draft of this paper, I had learned that in Russia, starting from the next school year, universities will have right to open and run specialist classes in mainstream secondary schools[†] – and this is happening in the country which already has an established network of specialist schools.

2.2. *Safeguards.* Running top level specialist mathematics classes and schools requires some safeguards.

- Teaching and learning have to be about enrichment, not "acceleration".
- Students should be invited to co-operate with each other, not compete.
- Specialist classes have to provide a safe psychological environment for students, and by all means avoid becoming "pressure cookers".

But discussion of these important points can wait until first practical steps are taken for setting up actual schools.

2.3. *Is it feasible?* Not every university will be willing to participate in the outlined scheme, but I can make an educated guess that perhaps half a dozen or a dozen schools with specialist classes around the country are feasible. This is a cheap and efficient way of improving the currently weak academic pipeline

school – university – MSc and PhD studies

in mathematics, physics, computer science, engineering. As the Green Paper emphasises,[‡]

> *Demand for higher level qualifications is growing strongly, and today's PhD students are often tomorrow's research leaders, entrepreneurs and industrial researchers.*

To have really good PhD students, we need first some bright secondary school students.

What possible harm could this modest proposal cause to mainstream mathematics education?

[†]`http://bit.ly/2mCWbEO`, assessed 04 March 2017.

[‡]HM Government, Building Our Industrial Strategy. Green Paper, January 2017, p. 29.

3. *Not just better, but different*

The reader who is interested only in policy issues can skip this section. It explains what is so special about specialist mathematics schools and answers the question in the title of this paper:

"What can specialist mathematics schools/classes give to students that mainstream schools cannot?"

As international experience proves, if schools are successful, they will enhance their students'

- ability to learn by absorption;
- ability to engage the subconscious when doing mathematics;
- ability to compress mathematical knowledge;
- capacity for abstract thinking.

Mathematics education theory and education policy discourse almost never mention these skills; I'll try to explain them point by point.

3.1. *Learning by absorption.*

In the sea of negativity around specialist mathematics schools, Professor Alison Wolf was quoted as describing one of their advantages:

> "*students find their tribe and learn from each other*".[†]

This is an aspect of mathematics / physics education of "mathematically able" children which is almost never mentioned: "mathematically inclined" (my preferred term) children have high capacity to learn by absorption. This trait remains dormant in normal school environment but gets activated when kids find themselves surrounded by children *like them*. My university has a large and vibrant community of mathematics PhD students, and it is a place where learning by absorption can be observed "in the wild". It is less known that the same could happen with a certain kind of 13–16 years old kids when they form a small learning community.

We need to remember that

mathematicians and physicists are stem cells of a technologically advanced society.

They have to be re-educatable, able to change their role, metamorphose – and inevitably have to be autodidacts in the process. Indeed, who will teach them in their professional future? They have to teach themselves and learn from each other. The key to success of a specialist mathematics school or class is the creation of a self-learning environment where students learn by absorption.

3.2. *Engaging the subconscious.*

Learning by absorption involves one more aspect of mathematical practice that is unknown outside the professional community of mathematicians.

[†] L. McClure, *All students should receive excellent maths teaching not just those in specialist maths schools*, `http://bit.ly/2k33Kj3`, posted 3 February 2017, accessed 18 February 2017.

I spent my life surrounded by people who got exclusive academically selective education in mathematics and physics, whether it was in the PhMSh[‡] in Siberia, or Lycée Louis-le-Grand in Paris, or Fazekas in Budapest, or Galatasaray Lisesi (aka Lycée de Galatasaray) in Istanbul – the list can be continued. I had research collaborators and co-authors from each of the schools that I mentioned. Why was it so easy for us to find a common language?

Well, the explanation can be found in the words of Stanislas Dehaene, the leading researcher of neurophysiology of mathematical thinking[†]:

> *We have to do mathematics using the brain which evolved 30 000 years ago for survival in the African savanna.*

In humans, the speed of totally controlled mental operations is at most 16 bits per second. In activities related to mathematics this miserable bit rate is further reduced to 12 bits per second in addition of decimal numbers and to 3 bits in counting individual objects. Standard school mathematics education trains children to work at that speed, controlling and verbalising each step: "left foot, right foot . . . ". Perhaps they can learn to walk slowly – but not many of them will ever be able to run.[‡]

By comparison, the visual processing module in the brain crunches $10,000,000$ bits per second.[§]

I offer a simple thought experiment to the readers who have some knowledge of school level geometry.

> Imagine that you are given a triangle; mentally rotate it about the longest side. What is the resulting solid of revolution? Describe it. [Answer is in footnote[¶].] And then try to reflect: where has the answer come from?

The answer comes from your subconscious. This is the best kept secret of mathematics: it is done by the subconscious. Moreover,

Mathematics, in one of its many facets, is a language for communication with subconsciousness.

If you were able to answer the question about the rotating triangle then you were able to pass your commands to the visual processing centers of your brain, which then managed to unambiguously interpret them and return you the result in the form ready for verbalisation and communicating back to me.

It is like training a dog.

Dogs have many faculties which we, humans, are lacking – for example, a fan-

---

[‡]A paper about this boarding school: A. V. Borovik, *"Free Maths Schools": some international parallels*, The De Morgan Journal 2 no. 2 (2012) 23–35. ISSN 2049-6559. `http://bit.ly/2jhXHcA`.

[†]S. Dehaene, **The Number Sense: How the Mind Creates Mathematics**. Oxford University Press, 2000. ISBN-10: 0195132408, ISBN-13: 978-0195132403.

[‡]Here I borrow some details from my book A. V. Borovik, **Mathematics under the Microscope: Notes on Cognitive Aspects of Mathematical Practice**. Amer. Math. Soc., Providence, RI. 2010. 317 pp. ISBN-10: 0–8218–4761–9– ISBN-13: 978–0–8218–4761–9. `http://bit.ly/2mrApka`.

[§]T. Nørretranders, **The User Illusion: Cutting Consciousness Down to Size**. Penguin, 1998, pp. 138 and 143. ISBN 0-713-99182-8.

[¶]Most people who I asked this question usually answered, after a few seconds of looking inside themselves, something like "*two circular cones glued at the shared base*".

tastic sense of smell. To exploit these faculties, we have to send our commands to the dog and interpret its reactions. A learner of mathematics is a dog trainer; his subconscious is his/her "inner dog" (or a puppy), a wordless creature with fantastic abilities, for example, for image processing, or for parsing of symbolic input. The subconscious has to be trained to react to commands "*triangle!*", "*side!*", "*rotate!*" in a way similar to a dog reacting to '*sit!*", "*bite!*", "*fetch!*"

How a learner of mathematics can achieve that? Perhaps even without noticing – in sharing his/her intuition with other likely minded young mathematicians.

3.3. *Learning to share intuition.* There are four conversants in a conversation between two mathematicians: two people, and their two "inner", "intuitive" brains.

When mathematicians talk about mathematics face-to-face, they frequently use language

- which is very fluid and informal;
- is improvised on the spot;
- includes pauses (for a lay observer – very strange and awkwardly timed) for absorption of thought;
- has almost nothing in common with standardised mathematics "in print".

Mathematicians are trying to convey a message from their "intuitive brains" directly to their colleague's "intuitive brain".

Alumni of high level specialist mathematics schools are "birds of a feather" because they have been initiated into this mode of communication at the most susceptible age, as teenagers, at the peak of intensity of their socialisation / shaping group identity stream of self-actualisation. Learning to speak to a peer's "intuitive brain" is an efficient way to learn language for communication with your own "intuitive brain". To facilitate this process of interiorisation of outward-directed speech (remarkably similar to the way toddlers learn to think by interiorising, directing at themselves their speech first directed at their parents and other people around them), students need a rich diet of challenging problems which go beyond application of procedural recipes, stimulate mathematical thinking, and require the use of deeper intuition and sharing of intuition.

In that aspect, mathematics is actually not much different from arts. Part of the skills that children get in music schools, acting schools, ballet schools, and art schools is the ability to talk about music, acting, ballet, and art with intuitive, subconscious parts of their minds – and with their peers, in a semi-secret language which is not recognised (and perhaps not even registered) by uninitiated.

3.4. *Compression and abstraction.* The specific modus of communication based on sharing intuition triggers the development of another mental skill specific for mathematics: compression of information. In the words of William Thurston, one of the greatest mathematicians of recent times,

> *Mathematics is amazingly compressible: you may struggle a long time, step by step, to work through some process or idea from several approaches. But once you really understand it and have the mental perspective to see it as a whole, there is often a*

> *tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compression is one of the real joys of mathematics.*[†]

In its turn, compression requires abstraction; I wrote about the strange fate of abstract thinking elsewhere[†]. Here, I mention briefly that it is hard to invent a concept more abstract than money – and money, as we all know, rules the world; for example, it is used in large scale and high frequency electronic transactions unpalatably quick for the human mind.

Children in their early teens are quite open to absorption of abstract concepts; after all, they are grappling with other important abstract concepts in their lives – for example, "love".

As the experience of specialist mathematics schools around the world shows,

use of mathematical intuition, ability to share intuition, compression, and abstraction

can be nurtured by uniting mathematically inclined students with their tribe, encouraging communication of mathematics, and providing children with rich mathematics, gentle academic guidance, and a strong value system. And it is proven in practice that this approach yields much more than an incremental positive effect expected from incremental improvement in education. Moreover, many university mathematicians know how to do that.

4. *Some political issues*

For some years I followed arguments around plans for top level specialist mathematics schools and started to feel that the idea was meeting resistance mostly on ideological and cultural grounds.

On the other hand, the need for such schools and classes is politically justified, but in a very different way: they offer a feasible solution to a serious strategic problem: nurturing and educating a generation of young people who will be able to serve their country in the Bletchley Parks of (alarmingly not so distant) future.

I heard this story in the pre-Internet age, and, as happens with most of the 20th century history, I found it difficult to check it using Internet resources – but I got it from a reliable source. In the Vietnam War, Russian military advisors to North Vietnam discovered that it was next to impossible to train fighter pilots from young men of communist Vietnam: because of malnutrition in their childhood, they had fragile blood vessels which could not withstand high G-forces and burst – and there was no way around this biological limitation. A future fighter pilot has to have a rich diet.

I have experience of working with "mathematically able" children, and I firmly believe that everyone has, at birth, a potential to eventually learn and understand

---

[†]W. P. Thurston, *Mathematical education*, Notices of the AMS 37 (1990), 844–850. The quote is on p. 847.

[†]A. Borovik *The strange fate of abstract thinking*, Selected Passages From Correspondence With Friends,1 no. 3 (2013) 9–12. ISSN 2054–7145. *http://bit.ly/2907Mmi*

mathematics, but this potential is gradually lost if proper intellectual stimulation and nourishment is not provided to a child.

In my understanding, one of the aims of "top level" specialist mathematics schools and classes is to provide some mathematical nutrients to some children in the mathematically malnourished country, in a hope that some of these children will become mathematicians, computer scientists, information security experts, electronics/photonics engineers, and cryptanalysts of the "Top Gun" class.

Of course, it would be much better to rebuild the entire system of mathematical education – but the costs of a total reform are unaffordable for this country in her present economic situation. Also, any measurable outcome of the reform will be seen only a generation later.[†] At that scale, a few specialist mathematics classes cost peanuts, but provide a limited, but well-focused and quickest possible solution, on the time scale of 5–6 years (when the first cohort of their students will reach the stage of postgraduate studies) – which is a blink of an eye in the glacial world of education policy.

Acknowledgements

This paper arises from my involvement with CMEP, the Cambridge Mathematics Education Project – I thank my CMEP colleagues, and especially Tony Gardiner and Martin Hyland, for many useful conversations – but neither they nor CMEP are responsible for my views expressed here.

Some material of Section 3 was developed for my talk at the conference *Mathematical Cultures I*[‡], organised by Brendan Larvor in 2012 with funding from Arts and Humanities Research Council under the 'Science in Culture' theme, with additional support from the London Mathematical Society.

Disclaimer

The views expressed do not necessarily represent the position of my employer or any other person, organisation, or institution.

About the Author

I am an research mathematician but I have 40 years of teaching experience at secondary school and university level in four different countries with four different education systems; since 1998 I am a Professor of Pure Mathematics at the University

[†]Socio-economic aspects of mathematics education policy are discussed in my paper A. V. Borovik, *Calling a spade a spade: Mathematics in the new pattern of division of labour*, **Mathematical Cultures: The London Meetings 2012-14** (B. Larvor, ed.). Trends in the History of Science. Springer, 2016, pp. 347–374. ISBN 978–3–319–28580–1. DOI 10.1007/978–3–319–28582–5_20. A pre-publication version (without editorial changes made by publishers): `goo.gl/TT6nc0`.

[‡]Mathematical Cultures, `http://bit.ly/2mSJO4g`accessed 05 March 2017. Proceedings volume: **Mathematical Cultures: The London Meetings 2012-14** (B. Larvor, ed.). Trends in the History of Science. Springer, 2016. ISBN 978–3–319–28580–1.

of Manchester. I teach a Foundation Studies (that is, Year 0) course in intermediate mathematics to 300+ students who were not successful in their A level mathematics, or did not take it at all, and for that reason I am on the receiving end of GCSE Mathematics.

I also have an interest in cognitive aspects of mathematical practice, please see my book *Mathematics under the Microscope*[†] which explains a mathematician's outlook at psycho-physiological and cognitive issues in mathematics and mathematics education, and touches on many issues raised in this paper. Some of my papers on mathematics education can be found in my personal online journal/blog *Selected Passages From Correspondence With Friends*[‡].

Email: ALEXANDRE ≫AT≪ BOROVIK.NET
Web: `www.borovik.net`; `www.borovik.net/selecta`

[†]A. V. Borovik, **Mathematics under the Microscope: Notes on Cognitive Aspects of Mathematical Practice**. Amer. Math. Soc., Providence, RI. 2010. 317 pp. ISBN-10: 0–8218–4761-9– ISBN-13: 978–0–8218–4761–9.

[‡]Selected Passages From Correspondence With Friends. ISSN 2054–7145. `http://www.borovik.net/selecta/`.