The De Morgan Gazette 9 no. 3 (2017), 11–16 ISSN 2053–1451

ANECDOTAL EVIDENCE

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1. *Computers take over*

In the last two chapters of [**2**], George Dyson retells a science-fiction story written by the late Swedish physicist Hannes Alfvén [**1**]. Alfvén wrote his tale in the 1960's with remarkable prescience. It is not a happy story. "Evolution on the whole has moved steadily in one direction. While the data machines have developed enormously, man has not." Computers take over the world, and the Great Disaster follows. Dyson writes:

> Those who had sought to use the power of computers for destructive purposes discovered that one of those powers was the ability to replace human beings with something else. What if the price of machines that think is people who don't?

Earlier in the chapter he also says:

> Alfvén's tale is now forgotten, but the future he envisioned has arrived. Data centers and server farms are proliferating in rural areas; "Android" phones with Bluetooth headsets are only one step away from neural implants; unemployment is pandemic among not working on behalf of the machines. Facebook defines who we are, Amazon defines what we want, and Google defines what we think.

Computers have not taken over the academic world yet, but we are already being subjected to new rules and restrictions of the algorithmic era. What computers can do will be done with all the power and finesse of quickly evolving technology; what they can't, will be banished to obscurity. Such is already the state of affairs in many areas, and in particular in education. What used to be a noble and delicate task of passing knowledge from a generation to a generation, is morphing into a set of simplistic learning goals and objectives, accompanied by standardized testing software. Successes and failures are measured by aggregated test results—one collects data, often electronically, and lets the software be the judge. Surely, there is always room for competing interpretations, as there are always doubts concerning data collection and the choice of methodology, but in the heat of politically charged debates such subtleties are easily overlooked; when it comes to public discourse and policy decisions, big data rules.

For those of you who are not familiar with the current trends in educational policies, I want to quote from a recent article [**4**] that I found online searching for information on evidence-based research on education:

> Throughout the history of education, the adoption of instructional programs and practices has been driven more by ideology, faddism, politics, and marketing than

2010 *Mathematics Subject Classification* 97A80, 97B70.

> by evidence. For example, educators choose textbooks, computer software, and professional development programs with little regard for the extent of their research support. Evidence of effectiveness of educational programs is often cited to justify decisions already made or opinions already held, but educational program adoption more often follows the pendulum swing of fashion, in which practices become widespread despite limited evidentiary support and then fade away regardless of the findings of evaluations. . . .
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> A key requirement for evidence-based policy is the existence of scientifically valid and readily interpretable syntheses of research on practical, replicable education programs. Educational policy cannot support the adoption of proven programs if there is no agreement on what they are.

It is clear that evidence-based research, mass-produced with the growth of computer technology, has become a standard practice in education research.

I am not writing this article to save the human world from computers (although I do worry about it), my concern is the teaching of mathematics. Despite enormous efforts of researchers and practitioners, the situation is bad. Liping Ma [**3**] gives a comprehensive picture of the reforms and changes that took place in the U.S. and elsewhere in the last 50 years or so, and concludes "... instability, discontinuity of teaching and learning, and incoherence among concepts have damaged U.S. elementary student learning. It seems that in the search for better methods of teaching, we lost the content of what is taught — mathematics itself."

I have been teaching at the City University of New York (CUNY) for over 30 years. I, and many of my colleagues, observe a constant decline in student preparation for college. Results of various standardized tests go up and down, but in our classrooms we see a steady decline. We often share anecdotes showing how confused our students are. The anecdotes may be entertaining, but the general picture that emerges is not funny at all. There is much that can be learned from those anecdotes, but this evidence is often ruled inadmissible in debates about the future of education. I suggest that we look for ways to admit them. A good anecdote is much more than just a good conversational piece. It is like a picture that is worth a thousand words. It may reveal something that might be completely overlooked in databases, no matter how detailed and complete. It tells some truth. It may not be a complete and universal truth, but a truth nevertheless. This truth should not be ignored.

Below, I present four anecdotes from my personal teaching experience. I am not making claims about their particular importance, but they seem very telling to me. Each may deserve a separate discussion, but I will not offer lengthy comments. They are like jokes, explanations kill them.

2. *The anecdotes*

There are a few students who get to college without knowing any mathematics at all. There are many more who have learned some mathematics, but unfortunately not of the right kind. Almost all students know the FOIL method, and the PEMDAS rule.[†] On trigonometry exams, many diligently write "sohcahtoa" somewhere on the

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[†]If you did not go to school in the US, you probably do not know what those terms mean. Google them.

exam paper, and often this is the only thing they do. Many students believe that π is $\frac{22}{7}$, and I have a very hard time to convince them that it is not. Many students do not know much more than that, but most of them, even if they remember more, it is almost always in a form of a recipe to "do" something. Very often they do not *understand* the reasons why the procedures they are expected to master work. Even worse, many are not aware that there is something to understand there. Teaching such students is difficult. To have any success at all, one has to learn about what the students know and how they know it. Learning this is not easy, because they do not know how to talk about mathematics. They almost never do it. To understand our students, we have to resort to indirect methods, and the "anecdotal evidence" helps.

2.1. *Solving quadratic equations.* I once made a presentation to high-school mathematics teachers about how we place freshmen in mathematics courses. I prepared a list of sample questions we ask during interviews. I usually begin by asking the student to solve an equation such as

$$(x+1)(x-2)=0.$$

I have done it many times, and on very rare occasions students produced the solution right away. Usually, they first expand

$$(x+1)(x-2)$$

to

$$x^2-x-2,$$

then diligently they factor the trinomial, and after that they find the solutions. When asked, why they proceeded this way, students usually say something like "I thought I was supposed to do it."

This anecdote is not about students, it is about the teachers. I was expecting a laugh, and an acknowledgement that something is missing in students preparation. I was going to give a few similar examples. But there were no laughs. Instead, the teachers became defensive. "What's wrong with what the students do?" they asked. "After all, they solved the equation, didn't they?" I skipped my other examples.

2.2. *The golden ratio.* What I will tell you about now happens every time I teach how to solve quadratic equations in an elementary algebra class, which means almost every semester. After reading, you may think that this happens in my classes because there is something wrong with the way I teach. I do not think so, but to be sure I have asked a few of my colleagues, whose teaching skills I greatly respect, to repeat my experiment in their classes. The result were the same.

After students learn how to solve quadratic equations, I like to discuss my favorite example:

$$x^2=x+1.$$

In class, we use the quadratic formula, write down all details and get the solution

$$\frac{1\pm\sqrt{5}}{2}.$$

While everything is still on the blackboard, I point at the equation and say: "Look. Let's think about it. Could there be a number such that when I square it, the result is the same as when I add one to it?" Consternation, silence. Then one or more students declare, that no, there can't be such numbers. I say: "Think again." "Zero," someone says. We check. No zero is not such a number. Then someone suggests that we try 1. This does not work either. They give up, and I say: "Look, there is a solution. Right here on the blackboard." "Where?" they ask. I point at the solution, and someone says: "You must be kidding!"

I am not kidding. This last reaction I only witnessed once, but every time this all happens according to the same scenario. No one in class seems to understand what we did when we solved the equation. Explanations follow, and we talk more about what is happening here. I learn much from students' responses. It seems absolutely clear that many of them do not know what it means to solve an equation, but that much I know already. I usually find it out earlier on the occasion of solving equations such as $x + 1 = x$. But now it turns out that even more advanced students are not aware that the two solutions they found are actually *numbers*! That was a surprise to me, especially since in the course, the previous topic we covered was simplifying radical expressions.

And here is a sub-anecdote. After discussing the equation and its solutions, I usually ask an additional question. I write the two solutions separately and ask which one is positive, and which is negative. Invariably, they are both declared positive. I know why students think that. I have asked them. Try to guess. The answer is in the footnote on the last page.

2.3. *Real mathematics.* The Discrete Mathematics course for computer science majors that I often teach, is designed as a bridge between intermediate algebra and higher level mathematics. My goal is to get students to think about justifications and proofs. For them it is a totally new experience. During the course, before any attempt to talk about mathematical logic and formal proofs, we discuss examples, many examples. They include proofs of irrationality of $\sqrt{2}$, the usual algebraic proof, and a beautiful geometric proof due to Stanley Tennenbaum; the proof that at a party with at least six people there are always either at least three people who know each other, or at least three mutual strangers; various proofs by induction; a proof that the set of rational numbers is countable, and the diagonal argument showing that there are uncountably many branches in the full binary tree. The pace is slow, allowing much time for all technical details together with a full discussion of what is being done and why. The success is never overwhelming—the students in this class are often those who in the previous semester struggled with

$$x^2 = x + 1.$$

Still, I see them making progress. Once, at the end of the semester, we had a discussion in class about the meaning of the work we did throughout the semester. Some said they liked and appreciated the class, but were doubtful of its value. One student expressed it this way: "Professor, we understand you, but look, we can not put too much work into this course, next semester we have to go back to a *real math class*."

2.4. *What math do we use?* I have many stories to tell, but recently, on social and professional occasions, I have been telling just one. This is my ultimate anecdote, and it is often met with disbelief. Again, what I am going to describe, did not happen once. It has happened many times in one form or the other, but this particular incident was particularly memorable. Basic Arithmetic is the lowest level remedial mathematics class that colleges offer. It is my strong opinion that there is nothing easy about it. When children learn arithmetic in school, it is done slowly and methodically, based on experience of generations of teachers, and it depends on building skills through a variety of learning experiences. Looking back, that material may seem very simple. After all, all kids learn it. It is not that simple though. Here is another quotation from Liping Ma [**3**]:

> One reason that U.S. elementary mathematics pursues advanced ideas is that the potential of school arithmetic to unify elementary mathematics is not sufficiently known. This is a blind spot for current U.S. elementary mathematics. One popular, but oversimplified, version of this trend is to consider arithmetic to be solely "basic computational skills" and consider these basic computational skills as equivalent to an inferior cognitive activity such as rote learning. Thus for many people arithmetic has become an ugly duckling, although in the eyes of mathematicians it is often a swan.

I could not agree more.

Basic arithmetic class that I teach includes some very rudimentary geometry. Students learn how to compute areas and perimeters of simple geometric figures. Nothing fancy: squares, rectangles, triangles, and L-shaped figures. You'd be surprised how difficult it is for students. A simple reason is that the students do not know what area is. Okay, one may think. I will just tell them what area is, and that will solve the problem. So this is exactly what I do in class. We draw rectangles, and, before multiplying their lengths and widths, we cut them into unit squares to compute the area just by counting them. I show how one can find the area of our classroom by counting the tiles on the floor. Then we use the same approach to compute areas of more complicated figures, and all seems fine.

At the end of the semester I review the material for the final exam. I draw a 2×2 square, and I ask the students to find its area. As expected, everyone knows it is 4. I explain one more time why it is so, by showing how the square can be cut into four unit squares. Then I erase one of those unit squares to make an L-shaped figure. "What is the area of this figure?" I ask, expecting a quick answer. Silence. "What is the area of this figure?" I ask again. "Seven?" someone says. "No," I say, "it is not seven." "Ten?" someone else tries. "No," I say, "it is not ten," and I get a bit frustrated. "So, what is it?" they ask. I say: "The area is three." Consternation. "How did you get it?" Now I really get frustrated. I go to the blackboard, and I slowly label the three unit squares saying: "one," "two, "three." A big man, who usually sits in the back, stands up and says: "Professor, we know that, but what math did you use! We need to show the math." What he wanted to tell me was that he was not going to get fooled by my simple explanations. What I was showing them was not math. For my students, math is what only very few mathematically gifted people understand, and what for everyone else is a collection of senseless rules and procedures that one has to show on exams for full credit. In my explanation, I did not show students any math.

I will let you draw your own conclusions.†

References

1. Hannes Alfvén, **The Tale of the Big Computer: A vision**. New York: Coward-McCann, 1968.
2. George Dyson, **Turing's Cathedral: The Origins of the Digital Universe**. New York: Pantheon Books, 2012.
3. Liping Ma, *A Critique of the Structure of U.S. Elementary School Mathematics. Notices of the American Mathematical Society*, vol. 60 no. 10, 2013.
4. Robert E. Slavin, *Perspectives on Evidence-Based Research in Education: What Works? Issues in Synthesizing Educational Program Evaluations.* Educational Reasercher, October 2016 (online).
5. *Evidence-based education.* Wikipedia, February, 2017.

About the author

Roman Kossak's research is in model theory of nonstandard models of formal arithmetic. Together with James Schmerl, he wrote a monograph on the subject, published in the Oxford Logic Guides series in 2006. For over 30 years he has worked at the City University of New York, where he teaches mostly developmental courses at Bronx Community College, and mathematical logic and model theory at the Graduate Center. His other interests include phenomenology, and interactions between mathematics and the arts.

†Here is an answers to the question: Why do students think that both solutions to $x^2 = x + 1$ are positive? Because there is no minus sign in front of them.