The De Morgan Gazette 9 no. 2 (2017), 7–10 ISSN 2053-1451

TUTORIALS FOR MATHEMATICS STUDENTS

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A previous paper, [**1**], has discussed tutorial work with large classes for engineering students. I would like to explain some trials I have made with different ways of interaction with mathematics students at different levels.

I heard that in the days of flush money with the UGC, Drama Departments maintained you could not teach drama with a student/staff ration worse than 6 : 1. By contrast, Mathematics Departments were saying that mathematics students could be lectured to in classes of 100, provided there were sufficient tutorials and example classes. However the role of tutorials was not much discussed.

Some regard a small group tutorial as another opportunity for the lecturer to explain things, which has the danger of not directly involving the students. Giving another mini lecture could be analogous to a piano student confessing to difficulties with a piece, and the teacher saying: 'It is easy, you do it like this'. This stresses that the teacher is much better than the pupil, which is hardly a surprise.

I liked to run a small group of 5 or 6 differently. The philosophy was that doing mathematics is something of a performance, or at any rate it is something one does, solving problems, writing, and so on, and so it was of interest to see how students went about this.

The aim of teaching could be said to move each learner from level l to level $l + \epsilon$. Here ϵ might be small, but iteration of this improvement could give large improvements. A journey can consist of one step after another. But one has to be clear on goals. Also experience shows that it is difficult to predict how quickly students will respond to a good training programme; in fact such a response, rather than measurement of level, is the best measure of differences between people. The starting point still has to be observation of level l, since this dictates action.

A psychologist friend very experienced in educational issues and learning theory told me that observation is a difficult art. Another of her points, based on lots of research, was that positive reinforcement is more important than negative reinforcement in any kind of teaching. Watching me teaching my handicapped boy a simple task she said: 'You are very good and clear about saying "No"', she said, 'but not so clear about saying "Yes". You should do it exactly the opposite way round' This was a useful lesson for a mathematician! I am not sure I have always followed it.

My method for a tutorial was that I would never write anything on the board. So a session would go something like this.

2010 *Mathematics Subject Classification* 97B40.

RB: Do you have any problems?
Class: Silence.
RB: Well what about the last example sheet. John, could you do question 3?
John: No
RB: Well come up to the board and have a go.
John comes up to the board with paper in one hand and chalk in the other. (This was some time ago, of course!)
RB: Any ideas?
John: No.
RB: Well, why not write down the question?
RB: OK. Now put the start of the question at the top and the conclusion to reach at the bottom of the board.

Thus the observation was that only too often a student may have no basic strategy for getting started. So my aim from my armchair was to guide John in a friendly way on how to get started, and continue as necessary, even dictating what should be written on the board.

Students should realise after a few weeks that the whole process was non threatening, as I was interested in what they could not do, in order to give them means to do it. The fact that they were writing helped them to learn the process.

Students also have to learn how to read what they have written, test each part, and rewrite it. I also tried another observation mode with first year students, with results which were interesting to me, though without long term evaluation.

I decided that one early first year assignment should be a 'gold' question, with possible marks of 0, 8, 9, 10 out of 10. So each student was handed one Part A A-level question, which on the marking scheme was awarded in A-level 6 or 7 points, a different question for each student. The students had first to hand this in. I then marked it with lots of corrections and a mark. The corrections were for grammatical style and also for mathematical accuracy. Thus sentences had to begin with a capital letter, and end with a full stop, the words 'Hence, Therefore, ...' had to be used correctly, and layout was important.

It was interesting to see that by and large the students could not write a clear account of even such a simple question. Moreover the questions did usually have a central mathematical point, which students usually had trouble in isolating, and making their answer clear, even when it was in essence correct. The process of handing back answers and getting revisions was allowed to go on twice, until 0 or a gold mark was achieved.

This whole process represented a lot of work, but I believe it did achieve the aim of conveying early on in their course what I saw as the standard for a decent piece of mathematical writing. I do not see other good ways of going about this. We are used to the idea of writing and rewriting, as part of our professional work. This is what we need to convey to students, of whatever standard of ability or speed of thought.

There is a radical school of medical thought which holds that 'Diagnosis without treatment is unethical.' I think there is an analogy here with 'assessment', which applies not only to our assessment of students but also to "higher bodies" assessment of us, in terms of teaching and research. Assessment is often seen as an end in itself, rather than a diagnostic tool for intended remedies.

A most rewarding experience of discussion with students came with the introduction of a third year optional half course called “Mathematics in Context”, [**5**], given with Tim Porter. This was started as an experiment, in conjunction with a course on the “History of Mathematics”. Members of the Department told us later they were sceptical about the course, but were amazed when the discussions between Tim and myself and six students went on for two hours. It was like opening the floodgates! Over the years the course became so popular we found it difficult to run. Also a typical “Maths in Context” question came from the students: “If you think ‘Maths in Context’ is important, why is it only an optional course in the third year?”

This led to trials of a first year course on “Ideas in Mathematics”, [**2**]. The concern addressed here was that first year courses tend be about teaching what are thought of as necessary skills for later years, but without any attempt at a picture of some things that are going on in the subject, and what contribution mathematics makes to the advancement of science and society. You can obtain a first class degree in mathematics without acquiring any sense that research is going on in mathematics, or of how the subject develops. This seems to be in contrast to other science subjects.

I found this course not so easy to construct, but what I did try out was some of the topics that I had given to a Masterclass for children in North West Wales. For example, one topic was “Higher dimensions”, ending with getting them to determine the number of 2-dimensional faces of a 5-dimensional cube. This caught the imagination of one student who eventually went on to get a PhD. Another topic I once gave was a problem about sharing bananas, or coconuts, among wrecked sailors, and a monkey, which was solved, some say by Dirac, but see [**9**], with the amazing start of minus 4 bananas. My intention was to illustrate how novel concepts can make apparently difficult things easy, and thus allow one to move on to more difficult things! This was also the course in which I tried out the “Gold Mark” exercise mentioned above.

In the end I settled on giving for this course a small chunk of Euclidean geometry, mainly on properties of circles and cyclic quadrilaterals, particularly Miguel’s Theorem. The advantage of this is its simplicity; the proof uses a theorem on angles in a cyclic quadrilateral, and its converse; and the fact that it can be developed to more complex situations and results. A further main reason for this choice was to get over the idea that the notion of proof is especially important when you have no reason to believe the result, i.e. there is the element of surprise. Then it is necessary to make the proof convincing. This conveys something of the spirit of mathematics, a feeling for which should be an element of a first year degree course.

Once in a first year analysis course a student wrote in an evaluation response: “Professor Brown puts in too many proofs!” So next year I decided the course would have no theorems and no proofs! It would, however, have “facts” and “explanations”, words which the students could understand. The requirement then was that the explanations should actually explain!

A reasonable general thesis is that the whole course structure and teaching should be designed with reasonable professional aims in mind. Compare for example the discussions in [**4**]. The book in which the article [**3**] appears starts its Introduction with: “No one disputes how important it is, in today’s world, to prepare students to understand mathematics as well as to use and communicate mathematics in their future lives."

One of the main points I want to make in this article is how avid so many of our students were for frank and open discussions on mathematics. One student wrote in an essay: "Lecturers think we cannot discuss mathematics: but they never see us discussing among ourselves." Somehow, this kind of tutorial discussion should be fitted into a University course, for the benefit of students and the promotion of mathematics.

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