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ACCELERATION OR ENRICHMENT?

SERVING THE NEEDS OF THE TOP 10% IN SCHOOL MATHEMATICS.

EXPLORING THE RELATIVE STRENGTHS AND WEAKNESSES OF "ACCELERATION" AND "ENRICHMENT".

REPORT OF A SEMINAR HELD AT THE ROYAL SOCIETY ON 22 MAY 2000.[0]

[For notes, see Section 6.]

ABSTRACT

This report was originally published in 2000 by the UK Mathematics Foundation[†] (ISBN 0 7044 21828). It was widely red, and was surprising influential. However, it appeared only in printed form. Various moves made by the present administration have drawn attention once more to this early synthesis— which remains surprisingly fresh and relevant. Many of the issues raised tentatively at that time can now be seen to be more central. Hence it seems timely to make the report available electronically so that its lessons are accessible to those who come to the debate afresh.

While the thrust of the report's argument remains relevant today, its peculiar context needs to be understood in order to make sense of its apparent preoccupations. These were determined by the gifted and talented policy' adopted by the incoming administration in 1997, and certain details need to be interpreted in this context. There are indications throughout that many of those involved would probably have preferred the underlying principles to be applied more generally than simply to "the top 10%", and to address the wider question of how best to nurture those aged 5–16 so as to generate larger numbers of able young mathematicians at age 16–18 and beyond. The focus in the report's title and subtitle on "acceleration" and on "the top 10%" stemmed from the fact that those schools and Local Authorities who opted at that time to take part in the *Gifted and Talented* strand of the *Excellence in Cities* programme were obliged to make lists of their top 10% of pupils; and the only provision made for these pupils day-to-day was to encourage schools to "accelerate" them on to standard work designed for ordinary *older* pupils. The wider mathematics community was remarkably united in insisting that this was a bad move. This point was repeatedly and strongly put to Ministers and civil servants. But the advice was stubbornly resisted; (indeed, some of those responsible at that time are still busy pushing the same line[‡].

The present administration seems determined once more to make special efforts to nurture larger numbers of able young mathematicians, and faces the same problem of understanding the underlying issues. Since this report played a significant role in crystallising the views of many of our best mathematics teachers and educationists, it may be helpful to make it freely available—both as a historical document and as a contribution to current debate.

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[†]The UK Mathematics Foundation, based in Birmingham, initiated a variety of modest initiatives in mathematics education during the period 1980–2010—including competitions, masterclasses, small-scale publishing, and professional development.

[‡]See `www.gtvoice.org.uk`.

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0. *Background to the report*

Recent government concern for the top 10% is welcome[1] [for notes see Section 6]. But the declared strategies (masterclasses, university summer schools, "world class" tests [available on demand "when pupils are ready"], early GCSE entry, etc.) tend to emphasize either

(a) unfocused (often non subject-specific) "enrichment"—lacking any sense of progression within a particular discipline, or
(b) "acceleration" within specific subjects.

Indeed

(1) LEAs participating in the DfEE *Gifted and Talented* programme are obliged to accept targets for early GCSE entries as a condition of their participation;
(2) one of the projects funded by the *Gifted and Talented* programme involves establishing *Advanced Mathematics Centres*[2] nationwide whose declared aim is to use early (Year 6!) GCSE entry as a focus for extra-curricular mathematics classes for able pupils in Years 5 and 6;
(3) while it remains unclear (both to schools and—apparently—to QCA, who are responsible for their development) what will be done to provide for pupils identified as "special" by "world class" tests at ages 9 and 13, it seems likely that the tests at age 9 may be used to identify pupils for the *Advanced Mathematics Centres*, and that those identified at age 13 may be seen as natural candidates for early GCSE entry in Year 9 or 10;
(4) schools applying for Technology College status are liable to be advised that "there is no officially declared policy, but early GCSE entry is now accepted [*sic*] as a way—which represents good practice [*sic*]—of demonstrating a commitment to the teaching and learning of able children".[3]

Hence one may be forgiven for highlighting "acceleration" as a significant thread in current DfEE policy.

These developments are part of a serious attempt to focus attention on the needs of able pupils— with £45 million allocated over 3 years.

It is not easy for those outside the DfEE to obtain a clear picture of what is being attempted, or why certain approaches are being adopted. It would appear that, instead of

building on what is known in this delicate area—where well-intentioned policies often have the opposite effect from that intended[4]—the *Gifted and Talented* strand of EiC (which we shall refer to simply as the "*Gifted and Talented programme*") has used the offer of money to develop policy on the hoof—openly adopting the slogan "*Ready, Fire, Aim*" [*sic*] to sum up this strategy.

In contrast, the seminar reported on here was concerned

- to explore realistic and achievable aims for the top 10–20%[1];
- to identify objectives and approaches which are likely to be *counter*-productive;
- to begin to clarify the appropriate target group for any national *Gifted and Talented* programme prior to implementation, and
- to indicate how this group of pupils might be better served.

That is, to identify desirable objectives [*Ready!*], and to analyse effective ways of achieving them which are consistent with existing provision for other pupils [*Aim!*], with the intention that this might then inform those devising pilot schemes for subsequent national policy [*Fire!*].

In *Excellence in Cities* [DfEE 1999; page 2]—immediately following the Foreword by Tony Blair and David Blunkett—we read:

> "To provide effectively for the aptitudes and aspirations of individual pupils we will: [. . .]
>
> - **extend opportunities for gifted and talented children** with special programmes for the highest performing five to ten per cent of pupils in each secondary school, including university summer schools and the introduction of "world class" tests".

At first sight these comments may seem to constitute general aspirations rather than rigid policy— especially since at the time of publication the DfEE had no experience of what might constitute a suitable "special programme", or a "university summer school", or a "world class" test. In the subsequent six months it became clear both

(a) **that there was—uniquely in the experience of most observers—a strong consensus on this issue among professionals concerned with *Mathematics* (including experienced teachers, educational psychologists, those in mathematics education, administrators, advisers, and mathematicians) as to the main pitfalls to be avoided**; and

(b) **that government policy was nevertheless being relentlessly developed without reference to, or acknowledgement of, this professional consensus.**

For example:

(1) When it expresses its misgivings, the government's own *Gifted and Talented Advisory Group* is repeatedly reminded that it is "purely advisory".

(2) At a recent meeting of the *Joint Mathematical Council*, which represents all sections of the wider mathematical community and which includes senior observers from government agencies, no-one managed to find anything to say in favour of "acceleration".

(3) Despite recommendations to the contrary, someone decided that there should be no overlap between the membership of the QCA Working Party which examined the implementation of "world class tests at age 9 and 13" and the *Steering Group* which was to oversee their subsequent development.

(4) The report of the QCA Working Party was—by agreement with the Chair of the Working Party—an open document; yet it was never copied by QCA to those who were subsequently appointed (by the same section of QCA!) to the relevant *Steering Group*; (and despite vague assurances to the contrary, we have failed to identify anyone to

whom the report was copied who might have been free to consider and to benefit from its analysis).

(5) It would appear that the *Steering Group* responsible for overseeing the development of "world class" tests has been kept less than fully informed of developments.

(6) Requests to the *Gifted and Talented* programme to support parallel pilot schemes which avoid the perceived pitfalls of "acceleration" (so that an informed judgement might be made at the end of the pilot programme) have been repeatedly parried, or simply ignored.

Academics, politicians, civil servants, and administrators operate under different "imperatives". The essence of democracy is that each should be free to ask awkward questions within the context of seeking to improve the quality, the value, and the impact of what is provided at public expense. This seminar was designed to explore (*in the public domain*, rather than within some advisory group whose hands and tongues are tied)

- **whether there exists (as many had suspected) a strong consensus**—*covering an unprecedented range of experienced professionals*—**as to the *in*advisability of some of the policies being adopted**,
- **with a view to making constructive suggestions in those areas where a consensus might emerge.**

A remarkably varied group of distinguished participants with a broad range of experience of studying, or of working with, able youngsters, met to consider (among other things) the relative strengths and weaknesses of "enrichment", "acceleration", and other strategies in encouraging the development of those who show a marked ability *in Mathematics*.

They also sought to identify the likely characteristics of an effective "national strategy" in the current English context.

They were therefore concerned to identify both

(a) the principles which should underpin official guidance and central action,

and

(b) the kind of central policy which might improve mathematical provision for the top 10%, and the practical support which teachers will need if they are to respond competently and flexibly within their own setting.

The report seeks to encapsulate the day's discussions. The participants' affiliations covered subject associations, experienced teachers, experienced end-users of talented youngsters, government departments, educational researchers, mathematicians, and those working on projects related to the *Gifted and Talented* programme.

Given that these participants came from different backgrounds (so that many did not know each other) and had different experiences, and since there has been relatively little professional discussion in recent years as to how one should best "serve the needs of the top 10% of able young mathematicians", participants were encouraged to submit short position papers for advance circulation: *five of these papers are included for information as appendices to the report* (Section 8).

Participants from government departments or agencies for the most part adopted the role of "auditors"—occasionally answering specific questions or commenting on statements of fact.

1. *Summary of main conclusions*

The following summaries are elaborated more fully in Section 4.

1.1. *Seven basic principles*

We give first a brief outline of seven principles which we suggest should guide any national strategy aimed at improving provision for able young mathematicians in English schools.

Focus: Provision for able pupils must have a clear focus and a clear sense of direction and progression if it is to have any tangible lasting benefit.

18+: The real test of any programme lies in the *future* attitudes and achievement of the pupils concerned. In particular, pupils benefitting from such a programme should leave school/college with greater competence in, and more commitment to, those subjects in which they have a special talent than would otherwise have been the case.

100%: Able pupils must learn to set themselves high standards. They should learn to insist on understanding what they are taught, and strive to get routine tasks completely correct.

16–18: The intensity and style of teaching and learning changes markedly in late adolescence. It is important, at age 16–18 when able young minds are ready to move up a gear, that they should be actively engaging with those subjects in which they demonstrate a marked strength.

Social: Exceptional talent inevitably draws attention to the individual possessing it. However, every adolescent needs space to develop away from the limelight[4]. Hence national policy should hesitate before *encouraging* strategies which remove individuals or groups from their natural cohort.

Provision: Yes. Identification: No! *Identification* is unavoidably exclusive and inflexible (How do you "de-identify" someone?); it is a poor predictor of future success; and it distorts the development both of those included and of those excluded. We need a model of provision which does not depend on prior identification (indeed, which recognises that appropriate *provision* plays a major role in the long-term process of nurturing— and hence of helping to identify—talented youngsters).

Institutions: Any national policy has to be delivered through institutions. So it is essential, when devising policies, to factor into the calculation the likely strain imposed on institutions by any proposed approach, and how this additional burden is likely to affect *other* students and staff.

1.2. *Acceleration: if in doubt, don't!*

On the basis of the extremely varied experience which participants brought to the meeting, there was a unanimous (and surprisingly strong) view

(a) that "acceleration" of individuals (in the sense of bringing forward work from subsequent years in order to keep pupils busy or extended) needs to be handled with great caution;

(b) that major acceleration often has serious disadvantages for pupils' long-term development, and can impose administrative costs out of all proportion to any likely benefits;

(c) that the only instances we could identify where acceleration might be appropriate were
- where the acceleration was *modest* and part of a *coherent strategy*—such as entering a *whole set* for GCSE a year early, when one has made sure that the mathematical and educational benefits gained by freeing an extra year for the students affected outweigh the possible disadvantages (e.g. the disadvantage of having pupils who have been accelerated, but whose grasp of key ideas remains fragile), or
- perhaps in a small number of exceptional instances depending on particular individuals and local conditions;

(d) that schools are currently tempted to use "acceleration" despite its evident drawbacks
- because it is an easy way of satisfying those who insist on "measurable", or "documented" outcomes (e.g. for OfSTED or DfEE);
- because of demands "that there be some tangible return"—made by governors or external agencies who provide additional funds;
- **because no other strategy is readily available**.

Thus, though "acceleration" may sometimes be justified locally, we could not see how it could form the basis of any national strategy.

1.3. *Enrichment for added breadth*

Enrichment for added breadth (such as getting pupils to work on number puzzles or logical teasers, or to construct hexaflexagons, or to cut up Möbius bands) has a long and honourable tradition. It is invaluable as an occasional part of some more serious strategy. However, it is too incoherent to form the main thrust of any national policy.

1.4. *Enrichment for added depth*

The remaining alternative is, rather than import additional topics which are unrelated to standard work,

> **to provide extension work *which enriches the official curriculum by requiring deeper understanding of standard material* (for example, by insisting on a higher level of fluency in working with fractions, or ratio, or algebra, or in problem solving).**

Thus, in line with our conclusions

(i) that "acceleration" should be avoided in most cases, and

(ii) that "enrichment for added breadth" cannot be the mainstay of any national strategy,

and in the light of clear evidence that "acceleration" is currently being used largely because official structures offer no viable alternative, we suggest **that there is an urgent need to open up a debate about curriculum structures, the provision of appropriate materials, and the kind of teacher support which might provide schools, teachers and pupils with a more satisfactory regime**.

1.5. *Teacher support*

We are concerned that no policy can succeed—especially in Mathematics—in the absence of

(a) carefully considered curricular (and extra-curricular) structures designed to help teachers meet the needs of able pupils,
(b) appropriate materials written with both teachers and pupils in mind, and
(c) suitably funded in-service opportunities for teachers to grapple with the mathematical and pedagogical issues relating to challenging able pupils.

1.6. *Short-term specific proposals*

A: We invite the *Gifted and Talented* programme to support the setting up of a *Working Group* to outline a possible curriculum structure detailing:
  (a) ways in which standard "core" material could be identified as inviting a treatment which develops "added depth", and
  (b) additional topics which are highly appropriate for the relevant age-group, but which do not pre-empt subsequent material from the standard curriculum.

B: We also invite *the Gifted and Talented* programme to provide the necessary additional funding to develop and run a pilot in-service programme targeting one mathematics teacher in each secondary school in one (EiC?) Local Authority—with a view to devising a strategy which might then be made available nationally.

2. *Setting the scene*

The renewed concern for the needs of able pupils was generally welcomed; but the failure to take note of what is well-known when formulating policy was regretted. Though the seminar was concerned with general issues, and did not wish to get involved in analysing specific pilot schemes within the *Gifted and Talented* (G&T) programme, we began with a survey of the range of activities and pilot schemes currently being funded, including:

- the current participation of 23 Local Authorities—rising to 44 in September 2000;
- the current secondary focus (to include some primary work from September);
- that each participating Local Authority has to appoint a G&T adviser, who is responsible for generating, coordinating and documenting local provision;
- that schools are linked into clusters covering all subjects;
- that funding goes to *clusters* of schools, but that each participating school has access to a (variable five figure) additional sum each year;
- that one coordinator in each cluster attends a four session national course which seeks to raise awareness of related issues; this single coordinator then has the task of passing on what s/he has learned to colleagues in *all* subjects in *all* schools within the cluster;
- that each secondary school in a participating Local Authority has to develop a G&T policy which addresses both in-school and out-of-school provision, and that each participating Local Authority has to accept "early GCSE entry" targets but can negotiate which participating schools "deliver" this Local Authority target;
- that each participating school is required to identify its "G&T cohort" of roughly 10% of the age-group (subject to central guidance and centrally decreed distinctions and proportions [70:30] between "gifted" [academic?] and "talented" [other?];
- that, while some schools tell pupils and parents when children have been so identified, many schools feel uneasy about doing so and design their extra provision to be inclusive of other pupils with the relevant ability whether or not they have been formally identified as belonging to the G&T cohort (these schools then refer to the nominated cohort only when returning their "accounts" to the central G&T administration);

- that the programme covers all subjects and all "talents", so that any distinctive provision for mathematics is left to individual schools, clusters and Local Authorities;
- that this has led to some interesting "general" provision, but without any clear focus or sense of direction; in particular, *the serious lack of coherently structured mathematical provision/progression is generally recognised* (though as yet nothing seems to be being done about this);
- that in 1999/2000 there were around 30 school-based "masterclasses" for selected pupils from a group of schools (rising, we were told, to 500 in 2000/2001), but that almost none had a clear mathematical focus;
- that there are to be 60 or so HE-based "summer schools" this year, with over 3000 places[5]—with the main type being week-long residential courses designed to provide a "university taster" for pupils in Years 11/12; the sessions are mostly discipline-independent, and many of those running the courses have no relevant experience[6] (e.g. a "summer school" for 100 students may be staffed mainly by 10 undergraduates and 10 PGCE students);
- that "world class tests" in mathematics and problem solving have been trialled and were run in June 2000 for around 10% of pupils aged 9 and 13 in volunteer schools[7];
- that the G&T programme includes a number of special projects—including the move to open *Advanced Mathematics Centres* in 25(?) centres nationally[8], where pupils in Years 5/6 will be given enriched mathematics classes on Saturday mornings and during the summer, partly to prepare them to take GCSE at the end of Year 6;
- that the DfEE is publishing materials (*Mathematical challenges for able pupils in Key Stages 1 and 2*), and that QCA and others were reported to be developing materials related to the Numeracy Strategy Extension (Years 7–9) and exemplary material for use with able children at lower secondary level.

"Acceleration" is clearly only one aspect of this flurry of activity. But, given the voluntary pronouncements of a number of key players, it appears to be a *significant* aspect, and represents perhaps the *only coherent thread*.

This impression has been reinforced by the only concern which has been openly acknowledged by those seeking to administer the current programme, namely the recognition that, when faced with the bewildering range of current schemes, teachers and others consistently ask: *What comes next? Where is this flurry of activity leading?* The question reflects the way policy is being created on the hoof, and the apparent underlying assumption that acceleration is good *per se*.

At present there appears to be no satisfactory official answer. Since nature abhors a vacuum, there is bound to be a strong temptation for administrators, teachers, pupils, and parents to reach for the easiest available options—namely acceleration and early GCSE entry (see Section 8.2)—and so to create (if inadvertently) an even worse vacuum in the mathematical education of these students at the crucial ages of 16–18.

3. *What we should be trying to achieve*

The seminar's morning session sought to focus on a general analysis of the issues and the principles which need to be considered when devising detailed policy. Nevertheless we agreed to bear in mind five questions which we had been explicitly asked to consider by the *Gifted and Talented* team from the DfEE.

1. What do we mean by "acceleration" and "enrichment"?
2. Can one base a national policy on "acceleration" and "enrichment" as *parallel* strategies?
3. Is "acceleration" wrong in principle in all cases—and if so, why?
4. If "acceleration" is sometimes acceptable, what are the circumstances?
5. What other conditions are needed for acceleration to be effective?

As the summary of conclusions in Section 1 indicates, when we moved on from generalities to specifics, there was a remarkable unanimity about the answers to these questions among participants (see Section 5). It would be unfortunate indeed if this unanimity was simply ignored by those in the DfEE who had specifically requested that we address these issues.

The discussion focused exclusively on Mathematics. This was not merely because most participants were mainly concerned with Mathematics. A very high proportion of those youngsters who stand out as being particularly able, stand out because of their particular strength in Mathematics. (For example, we were told that 85% of those identified on one database of very early children who took early GCSEs did so in Mathematics only!) This phenomenon of precociousness showing up more frequently and more markedly in Mathematics than in other core subjects is further reflected in:

- the fact that Mathematics is the only curriculum subject which features in the pilot "world class" tests, and
- the natural occurrence of Mathematics as a focus for the *Advanced Mathematics Centres*.

Clearly any national "gifted and talented" programme needs to pay particular attention to the needs of able young mathematicians.

We recognised that "acceleration" comes in many different guises. But we agreed to restrict the use of the word to refer to

- any strategy whereby individual pupils, or groups of more able pupils, are systematically fed standard curriculum work months or years ahead of their peers, thereby
  - (i) putting their learning (permanently) out of phase with that of their peer group, and
  - (ii) creating a potential vacuum—and the inevitable question "What next?" when the available standard work is completed before the age of 16 (in 11–16 schools), or 18 (in 11–18 or 13–18 schools).

We agreed to use the word "enrichment" to refer to

- any strategy which seeks to serve the needs of able pupils in ways which avoid (i) above.

We went further (see Sections 4.3 and 4.4) in distinguishing between

(a) "enrichment for added *breadth*", which supplements the standard curriculum with additional work which is more-or-less unrelated to standard curriculum mathematics (recreational topics, maths clubs, puzzles, etc.), and

(b) "enrichment for added *depth*", which routinely expects from those for whom it is appropriate
  - a higher level of technique in,
  - a greater depth of understanding of, and
  - a willingness to reflect on

  standard curriculum material, together with supplementary topics which lay stronger

mathematical foundations for subsequent learning in a planned and systematic way, but which do not pre-empt standard curriculum work from later years.

We considered the strategic question:

What should one be trying to achieve?

The *Gifted and Talented* programme has its own answer to this question, but it remains at the level of generalities:

> "To help improve the attainment and motivation of a cohort of 5–10% of gifted and talented children in each maintained secondary school—including under-achievers and pupils from disadvantaged backgrounds—by ensuring that they benefit from a coherent programme of in-school teaching and out-of-hours study that provides the right blend of challenge and support; and to improve the ability of all maintained schools to make effective provision for their cohort of gifted and talented children".

These are generally laudable aims; but it remains unclear what they mean in practice *for Mathematics*. Repeated attempts to find examples of provision *in Mathematics* have tended to emphasize the administrative gulf between the centre (where such laudable aims are drafted) and the trenches (where "coherent programmes" have to be developed). Moreover, the above general aims get mixed up from time to time with more elusive, and more questionable, official objectives—such as "the importance of involving such pupils in collaborative work".

It also remains unclear how the "coherence" and "effectiveness" of any of these programmes is to be evaluated. We were told that "Where targets are set, they are attainment-based"'; unfortunately, this again suggests such measures as counting "early GCSE entries"!

Among seminar participants there was general agreement that, from the specific point of view of Mathematics, the two most important principles were

(a) to ensure that all decisions considered the likely effects of any policy or action on the mathematical and social development of the pupils *in the long-term*, and
(b) to balance any benefit to individual pupils against the strain imposed on the school, on its staff, and on other pupils.

Short-term evaluations may help to identify, and hence to correct, administrative glitches. But they cannot demonstrate the effectiveness of a policy or regime—for there is no simple measure which can distinguish in *the short-term* between an effective and a flawed strategy[4]. Thus any serious evaluation of particular pilot regimes must take account of their long-term effects (at 18+ or 21). It is a mistake—thought a very common one—to imagine that there is some *a priori* gain in managing to accelerate pupils ahead of their age group; those who fall into this error are naturally inclined to think that long-term evaluation is unnecessary!

Thus our own answer to the question "What should we be trying to achieve?" is perhaps best summarised by the second, third, and fifth of our "Seven Basic Principles" in Section 1.1:

> **Any programme should have the effect that almost all of those who take part**
>
> **18+: leave school/college at age 18+ with greater competence in, and more commitment to, Mathematics than would otherwise have been the case;**
>
> **100%: have learned to set themselves the highest possible standards,**

should insist on understanding what they are taught, and should expect to get routine tasks completely correct;

Social: develop in a way which prepares them to make the most of their talents.

It is easy to see (just by waiting) whether the second of these objectives has been attained—even on the individual level. Assessing whether the first objective has been fulfilled is relatively easy on the statistical level—though the assessment of individual cases depends on professional judgement.

4. *Detailed comments on the main conclusions*

4.1. *The seven basic principles*

In this section we expand on the Seven Principles stated in Section 1, and contrast them with the general thrust of current policy.

Focus: If special provision for able pupils is to have any positive long-term effect, it has to have *a clear focus* and a *clear sense of direction* and *progression*. In particular, any national strategy has to start by developing appropriate *subject-specific* provision in key disciplines such as Mathematics.

[In contrast, most of the activities being generated by clusters of schools using G&T funds, and in "summer schools", are one-off sessions with no subject-specific focus, and so are likely to have no lasting effect.]

18+: The real test of the effectiveness of any programme of extra provision lies in the *future* attitudes and achievement (say at the age of 25 or so) of the pupils who take part. The minimum requirement for all such provision in schools and colleges should be that the pupils involved leave school/college with greater competence in, and more commitment to, those subjects in which they have a special talent than would otherwise have been the case.

[In contrast, many of the strategies being piloted are known often to have precisely the opposite long-term effect.]

100%: It is useful to distinguish between the *cognitive* consequences of any special programme and the *affective* changes which it succeeds in inculcating.

(a) In the purely *cognitive* domain, one expects most of those with a special talent for Mathematics to develop that ability further than their peers. Any special programme should seek to support this process by expecting more from such students than from their peers, and should therefore aim
  (i) to lay *stronger foundations*—including greater fluency in basic technique,
  (ii) to achieve a deeper understanding of that material, and
  (iii) to cultivate a greater willingness to reflect on connections between different parts of the discipline.

(b) In the *affective* domain, the subsequent development of able pupils depends on their having learned to set themselves high standards. In particular, they have

to learn not to be satisfied with second best, or with merely performing "above average", or with "getting the right answer most of the time".

In short, the most important lesson such pupils need to learn in Mathematics is

- not to be satisfied with incomplete understanding,
- to realise that one can (and should) *understand* the internal connections and reasonings which explain why a method is correct, and
- to insist on getting things completely right.

[In contrast, while we applauded the actual work done in the first *Advanced Mathematics Centres*, several participants expressed the view that the GCSE results obtained (see Section 8.5) were likely to teach pupils precisely the opposite of the "100% principle", and that this further supported the judgement that GCSE is an unsuitable focus for supplementary work with young children.]

16–18: One of the strengths of the English system for able pupils has been the way in which they have the opportunity to engage in depth with their strongest subjects after the age of 16. Even in countries with a broader "Abitur/baccalaureat"-style curriculum, the intensity and style of teaching and learning which is experienced by able youngsters changes markedly in late adolescence.
It is therefore important that, at age 16–18 when able young minds are ready to move up a gear, they should be actively engaging with those subjects in which they demonstrate a marked strength.

[Hence there is something wrong with any strategy which guarantees that those Mathematics courses which are intended to stretch able pupils aged 16–18 *are completed early* by the most able pupils, leaving them with no appropriate specialist alternative during their last years at school/college.]

Social: Though exceptional talent inevitably draws attention to the individual possessing it, every adolescent needs space to develop away from the limelight.

[Hence—though there are bound to be individual or local exceptions in which judgements are based on particular local conditions—any national policy should hesitate before encouraging strategies which remove individuals or groups from their natural cohort, thereby exposing them to public view and to the pressure of other people's expectations.]

Provision: Yes. Identification: No! To the administrator it may seem obvious that, before providing, one should first identify the target group. This would be reasonable only if two conditions were satisfied.

(a) First we should have ways of identifying talented youngsters which are sufficiently accurate to be *effective* ***predictors*** *of future achievement*.
(b) Second, we should be sure that the level of distortion imposed *by the process of identification itself* (on the development of those identified *and on those not so identified*) is acceptable.

In dealing with talented youngsters, neither condition appears to be satisfied! Hence the requirement for each school to "identify a G&T cohort" seems wise only if this is done tentatively, flexibly, and confidentially. (For example, it is unclear how to

"identify" openly in a way which allows a subsequent flow of pupils into and out of the "identified" cohort.)

The whole "identify-then-provide" approach is based on an application of the *medical model* (first diagnose, then treat) in a setting where both the diagnosis and the treatment are too poorly developed to make the approach appropriate. **We need a more flexible, more inclusive model of provision, which avoids the pitfalls of labelling certain pupils as *gifted*.**

Institutions: Any national policy has to be delivered through institutions. It is therefore essential

(a) to consider the additional demands which any policy seeking to improve provision for able pupils is likely to place on the affected institutions;
(b) to check that the demands made by such a policy do not undermine the ability of the institution to deliver other important (possibly *more* important) objectives;
(c) to balance the anticipated pay-off of such a policy against the costs (to other pupils, to schools/colleges, and to staff).

In particular, any attempt to improve provision for pupils with potential in mathematics

- has to address the dire shortage of well-qualified mathematics teachers, and to incorporate a realistic model for getting the best out of teachers already in post; and
- must avoid "solutions" which impose an extra administrative cost out of all proportion to any likely benefit.

4.2. *Acceleration: If in doubt, don't!*

There was a unanimous (and surprisingly strong) view

(a) that "acceleration" of individuals must be handled with great caution;
(b) that major acceleration often has unanticipated disadvantages for pupils' long-term development[4]; it also imposes a substantial additional burden on teachers and on schools, which diverts resources away from other pupils in a way that can be rarely justified by any net benefits;
(c) that—to repeat the conclusion stated in Section 1.2—the only instances we could identify in which acceleration might be appropriate were
   - where the acceleration was *modest* and part of *a coherent strategy*—such as entering a whole set for GCSE a year early, when one has made sure that the mathematical and educational benefits gained by freeing an extra year for the students affected outweigh the possible disadvantages (e.g. the disadvantage of having some pupils who have been accelerated, but whose grasp of key ideas remains fragile), or
   - perhaps in a small number of exceptional instances depending on particular individuals and local conditions;
(d) that many schools currently adopt "acceleration" because they find themselves under pressure (e.g. from parents, or from government) to act in an area where they have limited experience, and where they see no obvious alternative; (one participant who runs a centre for able children stated that 100% of teachers who adopted a strategy of "acceleration" said they did so because they saw no alternative, rather than because they thought it was the best strategy);

(e) that there was an urgent need to open up a debate about curriculum structures and teacher support which might offer schools and pupils a more satisfactory regime.

That is, though "acceleration" may sometimes be justified *locally*, we could see no way for it to play a major role in any coherent *national* policy.

There was also a strong view that any moves under (e) should **not** lead to "yet more tests": hence any new curriculum and assessment regime should avoid adding a new layer of demands, but should rather try to adapt (and if possible reduce the frequency of) existing tests. In particular, there appeared to be a strong feeling that we could not understand the dogmatic insistence that "world class" tests should take place *in the middle of Key Stages* at age 9 and 13, and that they would be far better if positioned as replacements for the "Extension" papers at the end of KS2 and KS3.

4.3. *Enrichment for added breadth*

In the last 30 years, "enrichment for added breadth" (weekly puzzles, maths clubs, masterclasses, occasional lectures, roadshows, etc.) has constituted the most common way of providing additional stimulus for able pupils. However, this preference reflected

(a) the lack of any national structures or funding regime which might have supported a more coherent provision incorporating some sense of mathematical *progression*;
(b) the fact that those making additional provision for able youngsters were generally operating as volunteers and/or as outsiders (e.g. from universities), and that the provision they were able to offer could never be more than informal and occasional.

Enrichment for added breadth has a long tradition, and is fine as part of some more serious strategy. However,

- it lacks any structure or focus which routinely transfers from one teacher to another—so provision of this kind often depends on happening to have' an exceptional teacher;
- the topics covered are in some sense "arbitrary" (in that different enthusiasts will choose different topics)—so that, while the enthusiasm conveyed is valuable, it is hard to see how provision by any *subsequent* enthusiasts can systematically build on earlier provision;
- many able pupils sense this lack of focus and progression and respond accordingly.

At the moment much of the out-of-school provision which seeks to "add breadth" is supply-led: for example, regions or Local Authorities with no local university are unlikely to have a local "masterclass". Any official schemes should try to develop a mode of provision which is demand-led, so that supply is determined by local need.

4.4. *Enrichment for added depth*

In line with the preceding remarks our judgement is clear:

(a) that "acceleration" is to be avoided in most cases, and should be at most occasional and modest depending on local circumstances and local judgement: hence it cannot be a serious part of any national strategy;
(b) that "enrichment for added breadth" has a role to play in any provision, but cannot be the mainstay of any national strategy (since it lacks a clear focus, and could never lead to a coherent programme with any sense of progression within the discipline).

The overall view was unanimously in favour if developing a strategy in Mathematics based on structures and programmes which exploit the principle of

enrichment for added depth

(without thereby excluding suitable material which "adds breadth"). However, if such a strategy

- is to be widely adopted,
- is to have long-term benefits, and
- is to change classroom practice,

then it is essential for it to have a clear *focus*—both in the sense of embodying mathematical progression, and by offering an appropriate pay-off (which may involve certification as a natural part of the standard national framework for all pupils).

There is therefore an urgent need to explore ways of devising, and making available, a coherent "extension" curriculum—designed, and constituting part of the standard provision for the most able 20% or so of pupils—which fits naturally alongside standard provision, which indicates the many topics and areas where teachers should expect deeper understanding of core material, and a few additional topics which supplement core material without pre-empting work from subsequent years.

Such a programme should perhaps be developed first for ages 11–16 (or 11–18).

We also need to ensure the provision of structured written materials, and *training opportunities* to prepare ordinary mathematics teachers to make the best possible use of any such extension curriculum.

4.5. *Teacher support*

At the risk of repeating previous remarks, we were very concerned that—in Mathematics in particular—no policy can succeed without

(a) carefully considered curricular (and extra-curricular) structures designed to help teachers to serve the needs of able pupils,
(b) materials written with both teachers and pupils in mind, and
(c) suitable (and funded) in-service opportunities to help teachers come to terms with the mathematical and pedagogical issues involved in challenging able pupils.

4.6. *Specific proposals*

A: The *Gifted and Talented* programme recognises the need for a specifically mathematical input. In line with point (e) in Section 4.2 and the comments in the subsequent paragraphs, we invite the *Gifted and Talented* programme to support the setting up of a *Working Group* to outline a possible curriculum structure detailing

(a) ways in which standard "core" material could be identified as inviting a treatment which develops "added depth", and
(b) additional topics which are highly appropriate for the relevant age-group, but which do not pre-empt subsequent material from the standard curriculum.
Despite concerns at primary level (and at A level—such as the need for a mechanism whereby *Further Mathematics* A level is available to all pupils), the consensus view was that one should concentrate initially on Years 7–11 (ages 11–16).

B: It was also suggested that the *Gifted and Talented* programme might provide the necessary additional funding needed to develop and run a pilot in-service programme targeting one mathematics teacher in each secondary school in one of the participating EiC Local Authorities.

Participants in such a programme could be expected to cover part of the course costs [though not the development costs] from their school G&T budget. However, since the Local Authority has no G&T money itself, and exercises no authority over its participating schools, the course would need to be attractively priced in order to involve "almost all" schools in the Local Authority.

To be effective, such a programme should be *short, intensive* and *pragmatic*. [For example, the programme could use good quality *pupil material* which is currently available as the vehicle for getting teachers to reflect on what it is that able pupils need, and how it can be achieved.]

5. *Additional remarks*

Most of the substantive comments from the day's discussion have been dealt with already. This final section presents the remaining substantive remarks.

5.1. *The five original questions on acceleration*

It is worth emphasising that we came up with clear, unanimous answers to the questions we were explicitly encouraged to address by the *Gifted and Talented* team in the DfEE (see Section 3.).

Question 1: What do we mean by "acceleration" and "enrichment"? This is answered fully—and as far as we could tell in a way that met with the approval of those officials present at the seminar—in Section 3.

Question 3: Is acceleration wrong in principle in all cases—and if so, why?

Some participants were at first inclined to answer; "Perhaps yes". But the eventual consensus view was that "acceleration" was acceptable only

(a) if it arose out of truly exceptional local circumstances, or
(b) if it was modest and controlled (as in entering a whole set for GCSE one year early, when one had a coherent reason for doing so and was in a position to ensure that the benefits outweighed the disadvantages).

[For further comments related to condition (b) see Section 5.2 below.]

The above answer to Question 3 also answers

Question 4: If "acceleration" is sometimes acceptable, under what circumstances is it acceptable?

and gives a partial answer to

Question 5: What other conditions are needed for acceleration to be effective?

This leaves:

Question 2: Can one base a national policy on "acceleration" and "enrichment" as parallel strategies?

Our clear answer was "No".

First "acceleration" is the easy, or lazy option:

- the necessary materials seem to be easily available;
- it appears to address the problem; and
- it produces an outcome which gets parents and administrators off one's back, and which rebounds to the teacher's credit.

But it is the *wrong* option (see Section 8.2).

Second, the outcomes of "acceleration" have a disproportionate appeal to politicians and the press, and to administrators (who can use them to issue yet more "targets", and to create new league tables).

Acceleration also has a strong appeal to parents and to pupils themselves (see Sections 8.2 and 8.5), and to school administrators and governors who are under pressure to trumpet their achievements within their local communities.

Where a school or Local Authority is willing to arrange special instruction for exceptional pupils, they often prefer to work towards early GCSE (sometimes as early as Year 5!) simply to provide an "end product" which might satisfy the educational bean-counters.

In all this, the *long-term* needs of the pupil get overlooked[4], and no-one ever returns to assess the eventual outcomes.

Thus the seductive appeal of precocious achievement (for parents, for pupils, and for teachers) is such that—in the spirit of *Gresham's Law*[9]—alternative strategies cannot compete. Hence any pretence that one can offer "parallel" strategies is likely to be an irresponsible fiction.

5.2. *Comments on current and proposed assessment*

Condition (b) in Section 5.1, which we suggest should be satisfied before engaging on modest and controlled "acceleration" (such as entering a whole set early for GCSE Mathematics), is more difficult to apply than might appear.

(b_1) Most schools are not in a position to enter a whole class for GCSE in Year 10, but may feel under pressure to enter a *handful* of individuals early. However, it is hard to ensure that such a small group is adequately prepared, so the syllabus may be covered only partly or superficially— sufficient to get a reasonable grade, but not to lay strong foundations for subsequent work.

(b_2) It is not easy—especially in an 11–16 school—to make good mathematical use of Year 11 if GCSE is taken in Year 10, For example, many schools encourage those who have taken GCSE Mathematics in Year 10 to take GCSE *Statistics* in Year 11, even though GCSE Statistics scarcely constitutes an appropriate *mathematical* diet for able pupils: hence, having taken GCSE Mathematics early, students regularly land up *doing essentially no mathematics for a whole year*!

In discussing the issue of what might constitute a more suitable mathematical diet in Year 11 for those who take GCSE in Year 10, there were several references to certain *Additional Mathematics* syllabuses, whose mix of "demanding material with restricted scope" was highly suitable for a one year course designed to deepen understanding. Our point is not to advocate any particular course, but to urge that serious thought be given to *ensuring that those who take GCSE in Year 10 face appropriate challenges in Year 11*.

(b_3) For those who by-pass *Additional Mathematics*, one common argument given for allowing able young mathematicians to take GCSE in Year 10 is to allow an extra year for them to take both *Mathematics* and *Further Mathematics* at A level. However, it is not easy for an 11–16 school to implement this strategy effectively, and current pressures make it impossible for most schools and colleges to offer *Further Mathematics* in a suitably flexible way. It is therefore important, as part of G&T provision in the area of Mathematics, to find ways of ensuring that Further Mathematics A level is available to **all** those who could benefit from taking it.

(b_4) In a similar spirit it was stressed that the current form of Mathematics GCSE serves the needs of the top 20% rather poorly. The weakness appeared to lie not only in the depth of understanding and the level of fluency demanded, but in the lack of multi-step problems to develop candidates' problem solving abilities.

Participants were unanimous in wanting (if possible) **fewer** tests; but they were also clear that suitable high quality assessments could feed back positively into the quality of teaching.

Questions were raised about the desirability of "differentiated papers in A level Mathematics" and about the possible inappropriateness of "modular" (as opposed to "linear / synoptic", end-of-course) exams for the most able pupils at age 18.

We were also deeply unhappy about the insertion of an additional layer of "world class" tests *between* the tests at the end of KS2 and KS2, and between the tests at the end of KS2 and KS3. Moreover, the kind of questions being rehashed as part of the pilot "world class" tests suggest that the final product is most unlikely to justify the label "world class". Thus Ministers would be well-advised to look again at the obvious alternative[10].

5.3. *Comments on alternatives to acceleration*

Several participants stressed that the most important objective should be to increase the frequency of ordinary good teaching, and so to minimise the need to intervene for a handful of able pupils. This remark may need some elaboration.

Every mathematics teacher (and every examiner) knows that syllabus coverage *per se* is not a reliable measure of progress'. (One participant described his recent surprise at discovering that his own Year 9 top set had covered significantly less of the syllabus than their peers: he claimed he had concentrated on teaching them *Mathematics*, and so had covered less of the syllabus than his colleagues. There are good grounds for taking his claim—namely, that he had been "teaching them Mathematics"—at face value[11].) This suggests the need to pinpoint ways of encouraging teachers to *expect more*—especially from their most able pupils—without necessarily covering the material faster.

The main problem would appear to be that, not only is it harder to teach well than it is to teach badly, but that existing structures make it even more difficult than it need be. Current policies tend to focus (public, administrative, and professional) attention on the "outcomes" of exams devoted to *simplistic one-step problems*. This puts pressure on teachers to think in terms of "training" and "hurdle-jumping" rather than teaching and learning.

One teacher emphasised the fact that there is plenty of suitably challenging material which fits naturally alongside the current National Curriculum. However, the time, opportunity, and encouragement needed to make use of this material were under ever-increasing pressure.

Another teacher stressed the danger of relying on "enrichment", and emphasised the need for a curriculum and for materials which could be used alongside ordinary classwork, and which encouraged and supported good practice.

A teacher from a comprehensive school indicated that the problem was much broader than the top 10%. At present, the practice of focusing mainly on the demands of the curriculum means (as OfSTED has documented) that many pupils "stand still" during Years 7 and 8. What is needed is a culture of steady progress—including the strengthening and deepening of material which has already been nominally "covered".

A participant with many years' experience in a comprehensive school expressed misgiv-

ings about a formal "extension curriculum", but was convinced that the pressures to cover material faster meant that even the best pupils were in danger of missing out on important mathematical experiences.

Another teacher was concerned that the expression "extension curriculum" should not be construed as meaning "optional". What was needed was a "coherent core curriculum for top sets" so that certain material (including some currently understood as being at Level 7 and Level 8) should be routinely taught to able students in Year 9.

All participants were aware that nothing could be achieved without an adequate supply of well-qualified teachers, and a programme to support teachers already in post.

The need to take a balanced view, and to assess any supplementary programme within the context of ordinary teaching, was expressed especially cogently by one teacher who pointed out that the challenge of seeking to provide for 10% of each age group was an enormous task, and that

> "for such a large group, their "maths club" **is** their ordinary classroom! If someone is able, the important thing is to be taught **well**. If we rightly want them to experience coherence, challenge, and progression in their mathematics, what we need to guarantee is good teaching."

He went on to suggest that a good part of the available funds should be spent helping to improve existing teachers and to recruit additional good teachers.

6. *Notes*

0. The seminar was made possible thanks to generous support from the *Royal Society* and from the *London Mathematical Society*.

1. The most appropriate percentage of pupils to target remains unclear. The current policy manages to be both too broad and too narrow, and may need some modification.

   (a) The official target is "5–10% of each cohort in each school". As a way of ensuring that every school develops a policy for its more able pupils, this makes excellent sense—even though the whole cohort thereby identified in one school might in fact be weaker than the weakest pupils in another school. In this respect, the official policy appears commendably broad-minded.
   (b) However, the figure of "5–10%" is so small as to guarantee that this will never be a whole class, so that any resulting provision will inevitably be based on occasional activities, or small "withdrawal groups"—which are a very inefficient way to use the available resources and can cause considerable disruption.

   If schools which contain a substantial number of able pupils are to do anything coherent for these pupils as part of their standard curriculum provision, there are reasons to suggest that they have to think in terms of something closer to 20% of each cohort.

2. The *Advanced Mathematics Centres* (AMCs) are modelled on *Grove Junior School* in Handsworth, Birmingham. There, the Headteacher, David Winkley, wanted to raise aspirations in the local inner-city community. He wanted to show local parents that pupils in Years 5 and 6 could both enjoy working at a higher level and *succeed*! Naturally he had no choice but to work within the pre-existing educational framework. He decided, in order to provide a focus for their work, and to validate their achievement,

that some of the pupils who attended the AMC classes would take GCSE (Intermediate tier) at the end of Year 6 (see Section 8.5).
One can only applaud the intentions of the original scheme, and the tenacity of those who made it succeed as an isolated initiative. However, when seeking to extend its scope from a maverick, local initiative to a national programme, it is important to preserve the *spirit* of the original scheme (by providing opportunities which catch the imagination of the local inner-city community, and which persuade some of them to entertain higher academic aspirations) while avoiding the temptation to imitate features—such as early GCSE entry—which may be inappropriate, unnecessary, or even harmful, in a national programme.

3. This is the *verbatim* advice given on the phone by an official from the relevant section of the DfEE (dealing with applications by schools for technology college status).

4. The current case of a 15 year old, third year mathematics undergraduate [Sufiah Yusof], is one of a long line of instances where particular universities have (through their long-standing and well-publicised practice of admitting such students) effectively misled ambitious parents—and maybe the DfEE—into believing that "encouraged acceleration" is a strategy without major drawbacks.

5. On this we were given contradictory official information. There appear to be two different kinds of HE-based "summer schools"—national and local. Both are targeted at social groups who have in the past missed out on higher education. We suspect that there are meant to be around 3000 places at summer schools of the first kind (restricted to those identified by their schools as being the most able), and perhaps 6000 or more places at summer schools of the latter kind.

6. We have for too long simply accepted the fact that certain sections of society are seriously under-represented among those who go to university. Thus one has to welcome *the idea behind* these "university-taster" summer schools. However, good intentions are not enough. The programmes which are being laid on in some HEIs appear strong on form but weak on substance, and may give a misleading impression of university.

7. Despite the media hype, the responses of pilot schools have often been strongly negative. In particular,
   (a) the *a priori* commitment to deliver part of the tests electronically—despite clear advice to the contrary—is creating serious problems;
   (b) the questions being set indicate that those developing the tests are taking advantage of the fact that the government has overcommitted itself on this front to recycle discredited material from the 1970s and 80s [perhaps on the grounds that, since to those running the scheme "any test is better than no test", they are unlikely to look too closely at the educational philosophy which underlies the questions being recycled].

8. That at least was the official plan! However, it would appear that, despite the financial carrot, Local Authorities have failed to take the bait. The number of centres volunteering would appear to be such that the pilot project has been given exceptional permission to operate some AMCs as centres for "enrichment" rather than "acceleration".
This should not be assumed to represent a change of heart! Those making the deci-

sions were well aware form the outset that the scheme ran counter to the judgement of those with relevant experience in the profession. They have consistently ignored both reason and experience; far from "seeing the light", the change would seem to be reluctant and purely pragmatic.

9. *Gresham's Law* originally referred to coinage in circulation: "Of two coins having equal legal exchange value, that of the lower *intrinsic* value tends to drive the other out of use". This law was formulated by Sir Thomas Gresham (1519?–1579), and has become an accepted rule-of-thumb in many spheres of activity. (The above precise version often gives way to the pithier paraphrase: *Bad money drives out good*.)

10. Namely that, since everyone involved knows that NCT Extension papers at KS2 and KS3 are thoroughly unsatisfactory, the "world class" tests pilot program should be adapted to form a potentially valuable replacement for these unsatisfactory "Extension" papers at ages 11 and 14 (rather than age 9 and 13).
Though some features of these "world class" tests have been quietly modified (such as the original idea that the tests should be available on demand—by pupils or parents—at any age), one should not assume that the modification proposed here will be easily accepted on the basis of its rational advantages. As Note 8 above indicates, further pressure may be needed before the inevitable is eventually conceded.

11. The school regularly succeeds in nurturing a large number of talented young mathematicians, and achieves remarkable results with a large number of pupils.

12. Tim Gowers (University of Cambridge) received a Fields Medal—the mathematical equivalent of a Nobel Prize—at the 1988 International Congress of Mathematicians in Berlin (as did Richard Borcherds—who is referred to briefly in section 8.1 and who is now at the University of California, Berkeley). Simon Donaldson was a Fields Medallist in 1986 and is now at Imperial College. [Note added 2012: Simon Norton is to some extent captured in the biography *The genius in my basement*, by Alex Masters (Fourth Estate, 2011).]

13. [Added in 2012: Ian Porteous died in 2011. But these materials are probably still available.[†]]

[†]See `http://www.maths.liv.ac.uk/lms/funmaths/`; `http://www.leeds.ac.uk/ace/TSA/FunMaths.html`; `http://www.nationalstemcentre.org.uk/elibrary/resource/3893/fun-maths-roadshow-activities`.

7. *List of participants*

Chair: Professor Chris Robson	School of Mathematics, University of Leeds; immediate past Chair of the *Joint Mathematical Council*; immediate past-President of the *Mathematical Association*; member of Council of *UK Mathematics Trust*
Secretary: Dr Tony Gardiner	School of Mathematics, University of Birmingham
Steve Abbott	Deputy Head Claydon High School; President *Mathematical Association*; Chair *Mathematically Promising Network*
Jack Abramsky	QCA
Toni Beardon	Department of Education, University of Cambridge; Director *NRICH*
Professor Margaret Brown	Professor of Mathematics Education, Kings College; member of the "world class tests" *Steering Committee*
Nigel Bufton	HMI OfSTED, Specialist Adviser for Mathematics
Tim Dracup	DfEE *Gifted and Talented* team
Professor Joan Freeman	Middlesex University; child psychologist specialising in giftedness'; member of the *Gifted and Talented Advisory Group*
Howard Groves	*United Kingdom Mathematics Trust*; Mathematics Teacher RGS Worcester
Professor Celia Hoyles	Chair *Joint Mathematical Council*; Professor of Mathematics Education, Institute of Education, London
Peter Jack	Head of Mathematics KE Camp Hill Boys School, Birmingham
Dr Valsa Koshy	Brunel Cente for Able Children; member of the "world class tests" *Steering Committee*
Peter Lacey	Chair *Association of Teachers of Mathematics*; Deputy Director NE Lincs LEA
Dr Gerry Leversha	Mathematics Teacher St Pauls School; involved in the work of the *British Mathematical Olympiad Committee*
Eileen McAndrew	OfSTED, HMI with special responsibility for *Gifted and Talented* provision
Lynne McClure	Research Centre for Able Pupils, Westminster College Oxford; involved in training for G&T coordinators
Dr Hugh Osborne	Director of Studies in Mathematics, Trinity College Cambridge
Dr Ian Porteous	Former Chair *Mathematics Education on Merseyside*
Professor Peter Saunders	Mathematics Department, Kings College; Chair *London Mathematical Society* Education Committee
Dr Stephen Siklos	Centre for Mathematical Sciences, University of Cambridge; many years of experience in dealing with mathematics admissions to the university
Hugh Taylor	*National Primary Trust*; runs the GCSE Saturday classes for Year 6 pupils at the two *Advanced Learning Centres for Mathematics* in Birmingham

8. *Appendices*

8.1. *The merits or otherwise of acceleration (W.T. Gowers[12], Trinity College Cambridge)*

This short essay consists of personal reflections, rather than scholarly thoughts about how to teach Mathematics in schools. I shall try to be brief. I can summarise my position by saying that I am fairly suspicious about accelerating young mathematicians except in the most extreme cases— and I do not count "being the best student the school has seen for 10 years" as extreme (since that happens in every school at least once every 10 years).

My own experience

Like most people whose lives have turned out satisfactorily, I have a tendency to think that the way I was brought up was the best way. But here, for what it is worth, is how I was taught mathematics at school.

I first started serious mathematics (i.e. beyond elementary arithmetic, tables, and so on) at prep school (King's College School, Cambridge). After a couple of terms I started usually coming top, and also finding myself bored by many of the lessons. I particularly remember learning how to factorise integers into primes—for the third time. By the time I reached the final year, I was given a certain amount of individual teaching by the wife of the Headmaster. She had read mathematics at Cambridge, so I was very lucky. I should stress that she was preparing me for scholarship exams and broadening my knowledge; she was certainly not trying to teach me what I would learn at my next school.

The preparation was successful and I went to Eton. The practise there was, and I believe still is, not to accelerate. Unlike at many public schools, everybody took three years (from the age of 13) to do the bulk of their O levels. I took O level maths after year 2 (which was standard), additional maths after year 3, A level in double maths and physics in a further two years, then seventh term Oxbridge entrance followed by a nine month gap (in which I did almost no mathematics) before going to Cambridge. In other words, my education took place at a leisurely pace.

Although I was sometime bored in maths lessons at Eton (which was inevitable, given that in mathematics there is always a wider range of ability), I was again lucky to have a very enlightened mathematics teacher—this time a former research fellow of King's College, Cambridge. He used to set us a weekly sheet of challenging problems not related directly to the A level syllabus. I remember not being made to really think seriously about A levels until perhaps midway through the second year of sixth form, at which point we started practising A level questions—discovering that we had, amongst other things, covered the necessary material.

The ethos there was very much that education came first and examinations second. And yet the examination results were very good. I am sure that this miracle was possible only because we were not accelerated, and I am very grateful for this. I feel I had not only a broad general education, but a broad mathematical one as well.

I should stress that, although I was good at maths at Eton, I did not automatically come top all the time, so there would not have been an overwhelming case for accelerating me even if the school had believed in so doing. (By contrast, Simon Norton[12]—who had been

at Eton a few years before me—could solve Oxbridge entrance questions from the day he arrived. He really was an extreme case, was taught separately and took an external degree from London while still at school.)

Other people I have seen

Occasionally students who are well ahead of their peers ask to go straight into the second year at Cambridge. The only example I know where this has not been a failure is Richard Borcherds[12]. (Actually, I can think of only two other examples. In both cases they did well to start with, and then gradually tailed off. Neither is now a practising mathematician, even though they were both extremely clever.)

Another obvious example of an accelerated mathematician is Ruth Lawrence. She is still a mathematician—at the Hebrew University of Jerusalem; but contrary to what many journalists expected, she seems unlikely to be the next Simon Donaldson[12]. One cannot know how she would have fared if she had not been pushed by her father. My guess is that she would have done at least as well, and would have had the benefit of a normal life.

Generalities

If I seem to concentrate only on the sort of mathematician who comes to Cambridge and who gets a very high first, it is not because I am interested only in these exceptional people, but simply because I believe that there is absolutely no case for even thinking about accelerating anybody else.

It is commonly believed that mathematicians' ability starts to tail off after the age of . . . ; well, estimates vary from the ludicrously low 25 or so, to about 40. I think that this may apply to a certain sort of freakish genius, which is positively correlated with eventual mathematical success, but not as strongly as is sometimes supposed. If somebody is like this, then they will probably teach themselves everything that their teachers know whether the school likes it or not. Even then it is not obvious to me that they should not be encouraged to broaden their education in other ways. People who come to Cambridge at the age of 16, say, are usually successful, but rarely right at the top of their year, and are rarely among the most successful in the end.

As for those who are merely very very good, I think they should be encouraged to do hard problems (as they are, for example, on the Mathematical Olympiad training course) and, again, to broaden their education—both mathematically and otherwise. There are some topics which are not taught much at university, but which would provide excellent background knowledge—such as Euclidean and projective geometry. There are other topics which are taught at university but are suitable for bright schoolchildren as well—such as elementary number theory (Fermat's Little Theorem, etc.), beginning group theory (up to, say, the identification of both groups of order six—I was shown *Lagrange's Theorem* at school), or beginning linear algebra (becoming fluent with matrix calculations, and more abstract definitions for the brighter ones). And speaking as a veteran of the process of selecting applicants partly by interview, I always greatly welcome those who can do routine calculations quickly and efficiently—things like integration by parts, curve sketching, and so on. This ability seems to me to be strongly correlated with more interesting kinds of mathematical ability—even though the tasks appear quite different.

8.2. *Acceleration or enrichment (Dr. Gerry Leversha, St Paul's School)*

The easy—but mistaken—option is to accelerate.

It is easy because it does not require any extra planning or insight: it is simply a matter of covering the same curriculum more quickly. The textbooks are already written; the assessment is already in place; the programme is familiar. It is easy partly because it appeals to parents and to the pupils themselves.

I have interesting evidence on this last point. A little while ago I was teaching two very able pupils who had already gained the twelve modules needed for Further Mathematics by the end of Year 11 and were planning to move on to Cambridge at the end of Year 12. This meant that I had one year in which I could teach them anything I liked—an opportunity which might appear almost ideal. I opted to cover Newtonian mechanics in some depth, using twenty year old textbooks and setting the sort of questions that were examined in the 1970s—questions requiring a synoptic understanding of both the pure and applied mathematics syllabuses, which were not broken down into bite-sized steps, and which did not make any bows in the direction of "relevance". My experience was dispiriting. It very soon became clear that my pupils had become addicted to "collecting modules", and were only really interested in covering as many as possible—despite the resulting fragmentation of the syllabus and the superficial coverage of important topics. This collecting urge may be one of the most basic human instincts—witness the current Pokemon craze, or the longer-lasting hobby of Munro-bagging'. But I was disappointed all the same. Even my best pupils seemed to be addicted to acceleration at the expense of depth.

The acceleration option is also tempting because it leads to an accumulation of credits, which looks good to external agencies such as OfSTED. There is an enticing political allure in being able to claim that pupils are "getting further quicker". But really it is a lazy option. What we should be doing with our most able pupils is exploring aspects of the existing syllabus in greater depth.

My work with the British Mathematical Olympiad has convinced me that there is plenty of material which can extend and challenge able pupils in "elementary mathematics". For example, we can focus on the difference between "real mathematics"—mathematics that requires a critical, rigorous, holistic approach—and what so often passes for mathematics at GCSE. We can try to develop skills in problem solving—skills which are by-passed by the standard syllabus and assessment. We can explore topics, such a pure geometry and elementary number theory, which have been sidelined over the years as being "too difficult", or "too pure". We can emphasize the difference between proof and conjecture. Much of this work does not actually introduce any more mathematical material than is already in the present syllabus. For instance, there is a modicum of geometry in GCSE—the circle theorems, congruent triangles, similarity—but it is approached at such a superficial level as to be dangerously misleading. Even our most able pupils can be forgiven today for believing that geometry is a matter of *calculation* rather than *demonstration*. The syllabus makes bows in the direction of certain number theoretic concepts—prime numbers, rationality, irrationality—but without any requirement of depth of understanding. Other vital mathematical tools are overlooked or treated inadequately—with serious consequences for subsequent work. For example, when rational functions are treated (towards the final year of an A level course), most students cannot handle the algebraic operations involved; this clearly shows the lack of preparation in earlier years—in this case going as far back as the failure to insist on mastery in working with algebraic fractions. At the same time there are plenty of topics already within the syllabus which serve no useful purpose at all and which

would be best left for a proper treatment at A level (one which springs to mind is that of Standard Deviation).

I suspect that we need to de-celerate our most able pupils if we want them to become mathematically competent.

8.3. *Variations on enrichment (Dr. Ian Porteous, University of Liverpool and Mathematics Education on Merseyside)*

My experience of working with able youngsters is based on

- over twenty years' work developing the local MEM (*Mathematics Education on Merseyside*) Challenge and Senior Challenge competitions;
- ten years' experience of Masterclasses (not only in Liverpool);
- two years' experience developing the *FunMaths Roadshow* of the Liverpool Mathematical Society; and
- one year's experience of running a Saturday morning maths club for youngsters with a craving for mathematics.

I have also had close involvement over the last six years or so with several able youngsters from around Merseyside who have gone to Cambridge, having perhaps taken GCSEs and A levels I Mathematics early, but who—with one notable exception—have not left school early, but who have continued to develop their talents in other ways. (For example, one lad requested permission to attend a second year university course on Real Analysis while still in the Sixth Form; another took on the task of rewriting the manual for first year engineering students so as to introduce them to "C" rather than to Pascal.)

As a matter of policy, I am all for enrichment rather than acceleration; but if a youngster insists on taking exams early and is clearly way ahead of his or her year, then I do not think s/he should be prevented from so doing. However, what really matters is to consider carefully what happens long before this question can arise: *How is the interest of such students awakened in the first place*? And what can we then do to help them to develop healthy attitudes to their current and future work in Mathematics?

My starting point is with the Liverpool Mathematical Society *FunMaths Roadshow*. This is designed to give everyone a measure of success—presenting maths not only as a challenge, but as an *enjoyable* challenge. The current material is aimed at top primary and lower secondary pupils, but we have recently received a COPUS grant to extend the material for use by pupils in Years 9 and 10 (and the relevant boxes "5" and "6" are almost ready for trialling[13]). During the academic year 1998/9 the Roadshow was taken to well over 100 venues in celebration of the centenary of the Liverpool Mathematical Society. An important feature of the way the Roadshow operates when it visits a school is that the best support staff are often talented older pupils in the same school, who get a tremendous kick out of the chance to share their own enthusiasm for maths with younger pupils. Moreover, this enthusiasm seems to be accepted by their peers, and sometimes rubs off on them also.

Masterclasses are generally aimed at pupils in Years 8 and 9, and raise the perennial question: "What next?". Here is a strong demand from a small number of boys and girls for something in subsequent years—they really have a craving for Mathematics. This is how our once-a-month *Saturday Maths Club* was born. Here no holds are barred. The emphasis is on providing substantial enrichment by getting the students to grapple with genuinely mathematical problems and theories, and giving them the chance to get to know—and to work alongside—active research mathematicians. This is not coaching for premature A

level entry (though some of the topics covered may help to underpin A level work, and so improve students' performance as and when they come to take the exam).

8.4. *Serving the top 10% in mathematics (Dr. John Smith, Head of Mathematics Winchester College)*

1. Able students need a structured, coherent and demanding programme of study appropriate to their abilities. They should be in largish classes with students of similar ability, which meet regularly in exactly the same way that other classes meet. They need well-qualified and stimulating teachers.
2. Most students who show high ability in Mathematics are not dedicated to the subject. The majority are academically motivated and ambitious to succeed in life; they respond to good teaching; but one should not assume that they are wedded to, or motivated by pure love of, the subject. Healthy children with a variety of interests are likely to have other (*they* might say "better") things to do with their spare time than attend voluntary sessions in Mathematics!
3. The very able (and many not so able) students are of course stimulated by competitions and "special things" like Saturday morning classes, which can have much to offer them. However, we are deluding ourselves if we think that these activities play a major part in the education of the relatively broad 10% that we are considering.
4. The idea of giving specialist teaching to the most able students in state schools has been deeply unpopular for a generation. In a comprehensive school, where there are children who cannot read and write, it may seem unfair to dedicate valuable resources to top maths sets (or to Latin!). Thus the top 10% have not fared as well in recent years as in the old grammar schools. They may fare better in good sixth form colleges, but that may be too late to encourage a student to choose to take A level Mathematics, for example. Sixth form colleges also divert able mathematics teachers (who are always in short supply) away from other schools, so that bright students are less likely to experience such teachers in their formative years.
5. Public exams should never dominate teaching: I prefer not to talk about an exam until six months before a pupil has to take it! But exams and syllabuses have an important part to play in setting standards and goals. At present we have differentiated standards at GCSE (and in each set of Key Stage tests); so it remains a mystery as to why there are not similarly differentiated papers at A level for the aspiring A or B grade candidate. (I doubt whether the new Advanced Extension Award papers will contribute much unless a good number of universities feel able to require them. This would be quite different if no candidate could achieve an A or B grade without taking—and performing adequately on—an "Extension" paper.

8.5. *Advanced Mathematics Groups (AMGs) (Hugh Taylor, National Primary Trust)*

Background: We started our first AMG in 1997 at *Grove Primary School* (Handsworth, Birmingham). Prior to 1997 Grove had run a project whose purpose was to enrich the mathematics curriculum, with small numbers of pupils being entered for GCSE on an *ad hoc* basis each year.

We wrote to other primary schools in the area and invited them to send suitable pupils for Saturday morning classes—held under the aegis of the National Primary Trust. In this

way we managed to involve a good number of local schools and attracted an interesting group of able youngsters.

The aim of the AMG was to meet the needs of primary aged children who found Key Stage 2 mathematics a very meagre diet. As the focus for our work we chose to offer a GCSE course for Year 6 pupils. The perceived advantages of this were:

- GCSE is a well-established course of study;
- it provided a focus for the children's work;
- primary pupils seemed to gain a psychological boost from doing what they perceived to be work usually done by older pupils;
- the teachers involved found it easy to relate to the GCSE focus.

Few primary schools have staff who are familiar with GCSE work—so we decided to staff the AMG with one primary and one secondary teacher. We also made some use of sixth form students form King Edwards School, Aston—a move which proved popular both with them and with the primary pupils.

Outcomes: The following observations are based on nearly three years of running an AMG.

- With few exceptions (e.g. trigonometry, standard form) Year 6 pupils cope easily with the content of an Intermediate GCSE syllabus.
- One year of Saturday classes alone is not sufficient to do all the necessary work for GCSE in a satisfactory manner.
- About three-quarters of the pupils gain a grade D or better (see the table below).
- The pupils clearly enjoy the sessions (the drop-out rate is only about 5%) and are willing to work incredibly hard.
- We only use calculators when absolutely necessary. Number skills are practised regularly and are very good; work with number patterns is very popular.
- We point out children's mistakes, but don't dwell on them.
- Primary children are prepared to try almost anything. They are unfazed by the idea that a topic may be "too hard" for them. They are incredibly receptive and are a joy to teach.

Reflections: Last September I started a second AMG—based at *Moat Farm School*, Sandwell. Moat Farm School—like Grove—is far removed from the leafy suburbs! Unlike Grove, the class at Moat Farm does not have an age limit, and to my great surprise, children as young as 8 seem to cope well with the work set. As in Grove, the Moat Farm AMG is staffed by myself (an experienced secondary teacher) and a local primary teacher. This seems to be an effective combination.

We get a large number of visitors at both sites—but far more interest is shown by those from primary schools than by those from secondary schools. We have even experienced a degree of hostility from secondary schools; this is a cause for concern and we do not yet have a solution. At the moment we have no method of tracing our pupils' progress when they move on to secondary school.

I am not convinced that a series of unrelated "masterclasses" is most appropriate for primary children. The AMGs offer a year-long course, with continuity of staffing and the associated benefits: any alternative would have to be comparably coherent, and relatively easy to organise and to teach. I am convinced that the AMGs constitute a valuable contribution to primary education— though it remains to be seen whether GCSE is the best way of providing the necessary mathematical focus.

It would be interesting to extend the work of the AMGs to secondary schools. This would need the cooperation of a group of secondary schools, since a single comprehensive school is unlikely to have a large enough talent pool to make such a scheme viable. Since a large minority of likely participants may already be at the level of GCSE Grade C or above, a GCSE focus is likely to be inappropriate. There are a number of other possibilities—including modular A level mathematics.

Results (SEG GCSE)

June 1998 (SEG GCSE *Intermediate*)

Grade	**B**	**C**	**D**	**E**	**U**	**Abs**	**Total**
	2	3	6	2	4	2	19

June 1999 (SEG GCSE *Intermediate/Foundation*)

Grade	**B**	**C**	**D**	**E**	**U**	**Abs**	**Total**
Foundation			5	3			8
Intermediate	5	11	11	4	2	1	34